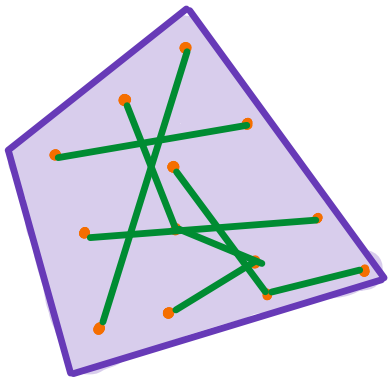
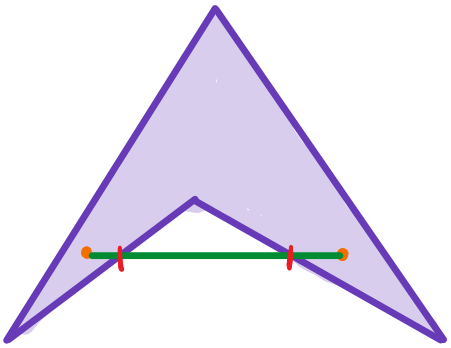
QUADRILÁTEROS

DEFINIÇÃO

QUADRILÁTEROS SÃO POLÍGONOS COM 4 LADOS



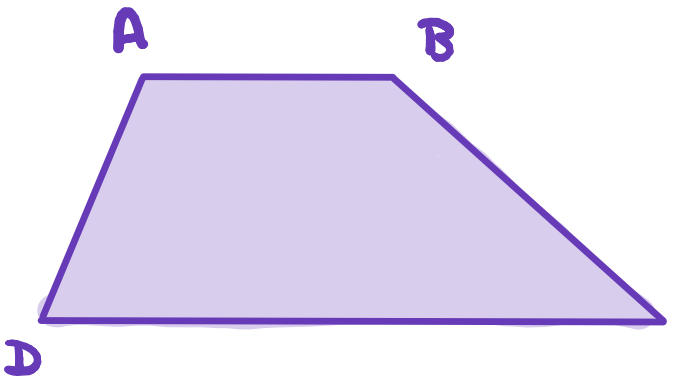
CONVEXO



NÃO CONVEXO

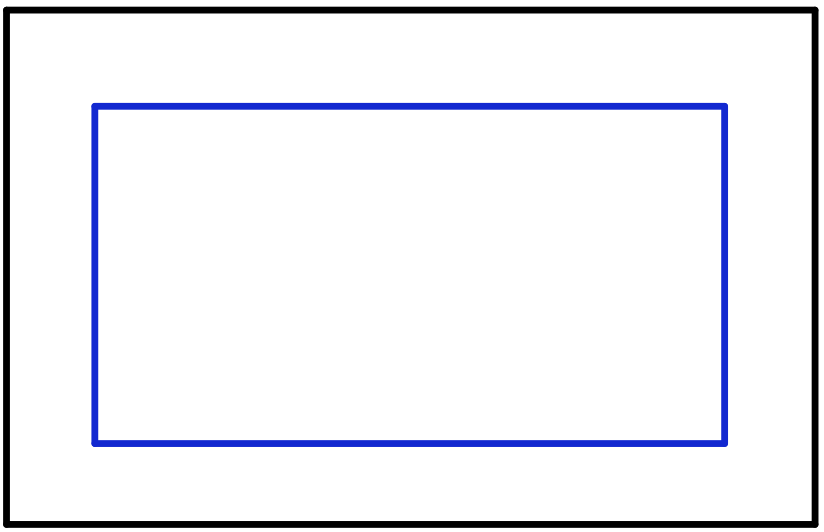
TRAPÉZIO

TRAPÉZIOS SÃO QUADRILÁTEROS CONVEXOS QUE POSSUEM DOIS LADOS PARALELOS, DENOMINADOS BASES.



AB // CD : BASES

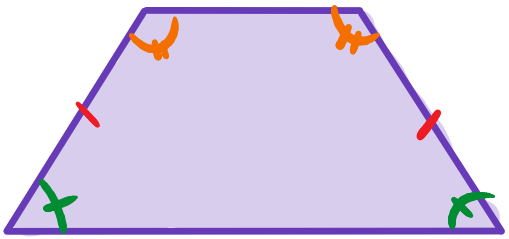
AD, BC : LADOS OBLÍQUOS



- QUADRILÁTEROS
- TRAPÉZIOS

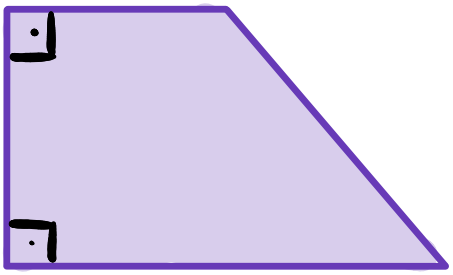
TRAPÉZIO ISÓCELES

LADOS OBLÍQUOS SÃO CONGRUENTES.



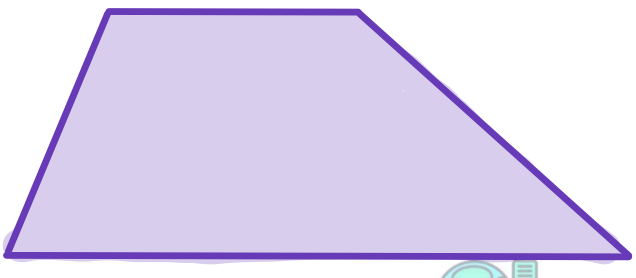
RAPÉZIO RETÂNGULO

LADO OBLÍQUO PERPENDICULAR À BASE.

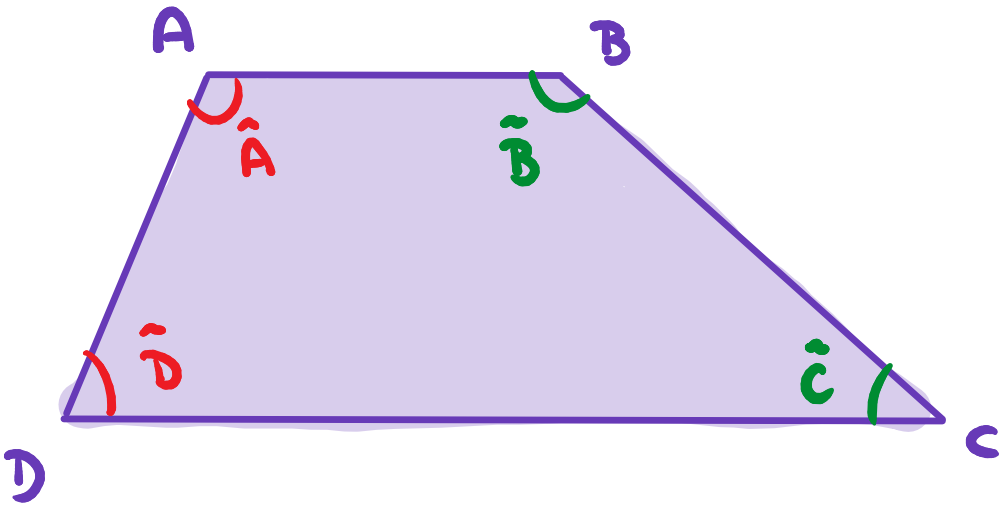


RAPÉZIO ESCALENO

LADOS OBLÍQUOS DIFERENTES



ÂNGULOS

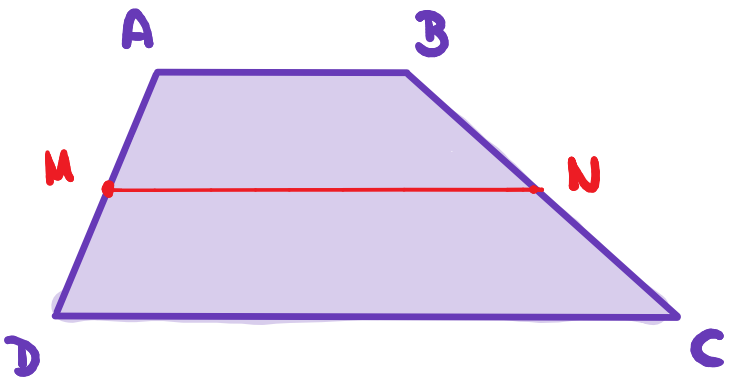


$\hat{A} + \hat{D} = 180°$

$\hat{B} + \hat{C} = 180°$

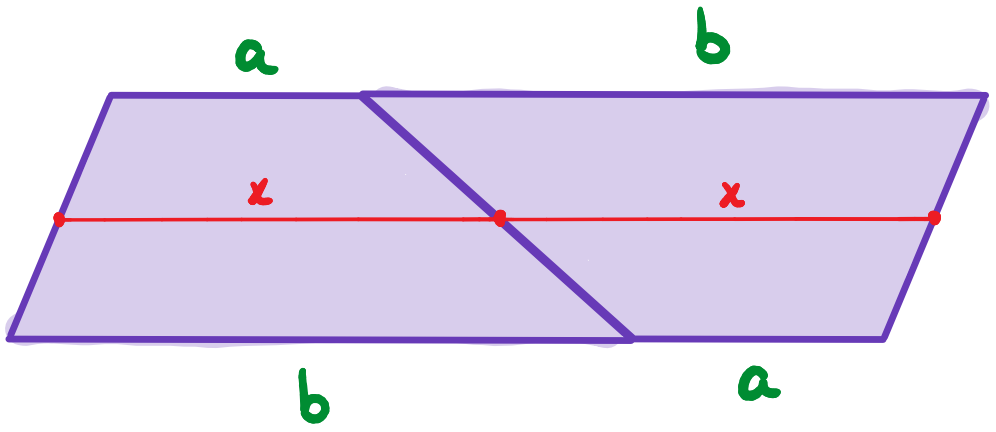
BASE MÉDIA

SEGMENTO QUE LIGA OS PONTOS MÉDIOS DOS LADOS OBLÍQUOS.



$\overline{MN} \parallel \overline{AB}$

$\overline{MN} \parallel \overline{CD}$



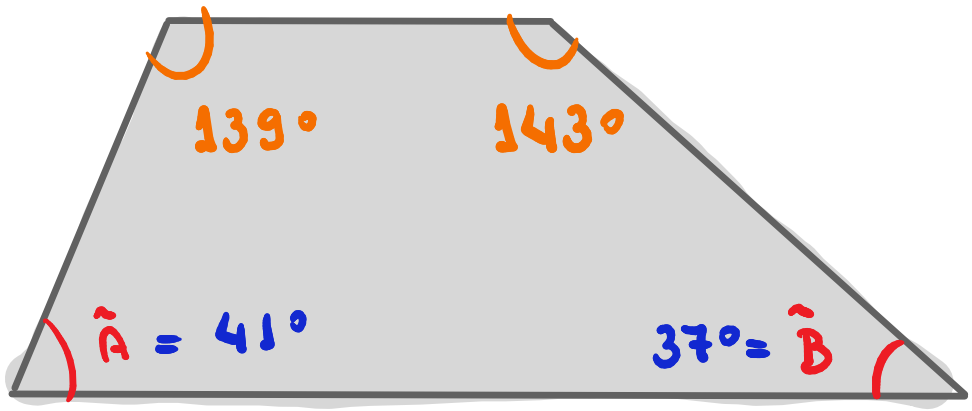
$$2x = a + b$$

$$x = \frac{a + b}{2}$$

EXEMPLO

A SOMA DE DOIS ÂNGULOS DE UM TRAPÉZIO É 78° E A DIFERENÇA ENTRE ELES É 4°.

DETERMINE A MEDIDA DO MAIOR ÂNGULO DESSE TRAPÉZIO.



$$\begin{cases} A + B = 78° \\ A - B = 4° \end{cases}$$

$$2\hat{A} = 82$$

$$\hat{A} = 41°$$

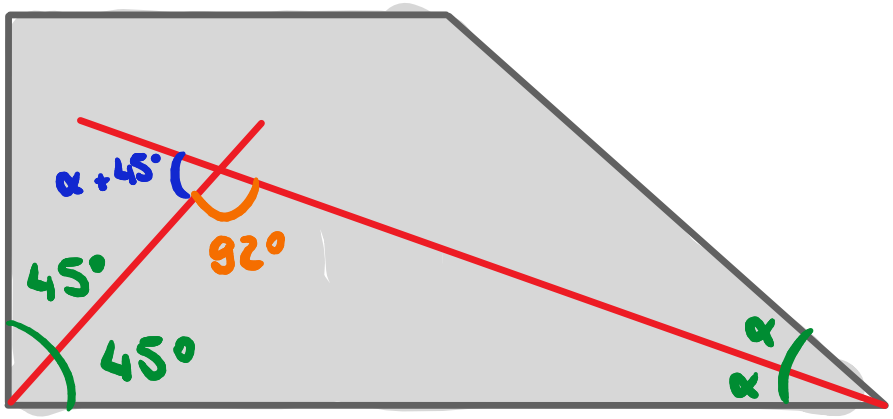
$$\hat{B} = 37°$$

143°

EXEMPLO

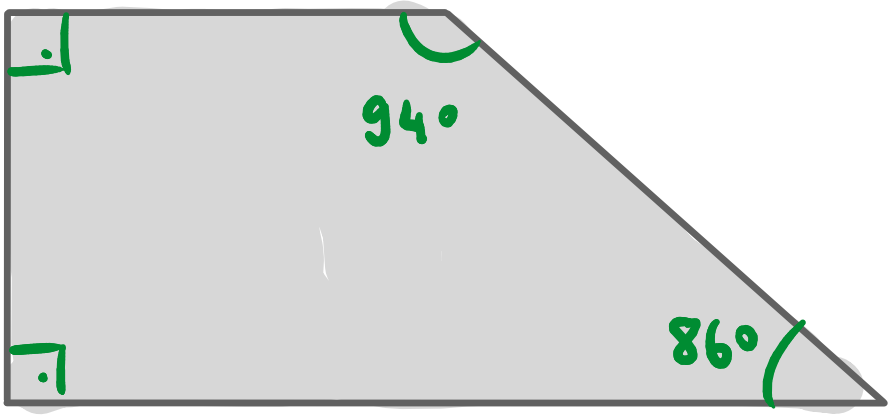
SEJA ABCD UM TRAPÉZIO RETÂNGULO. O ÂNGULO FORMADO PELAS BISSETRIZES DO SEU ÂNGULO RETO E DO ÂNGULO CONSECUTIVO DA BASE MAIOR MEDE 92°.

DETERMINE A MEDIDA DOS ÂNGULOS DESSE TRAPÉZIO.



$$\alpha + 45° + 92° = 180°$$

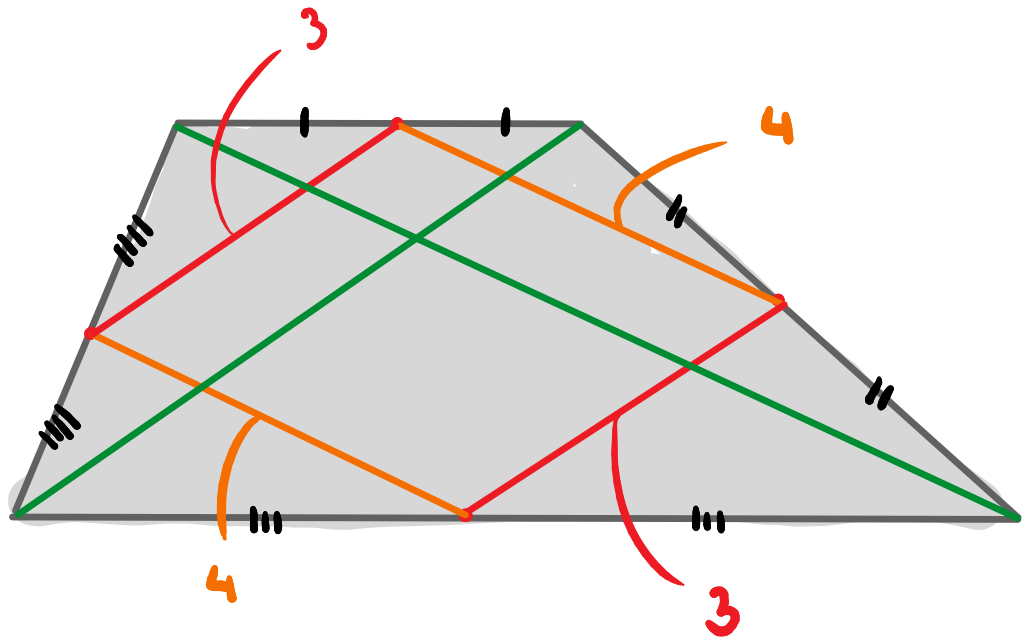
$$\alpha = 43°$$



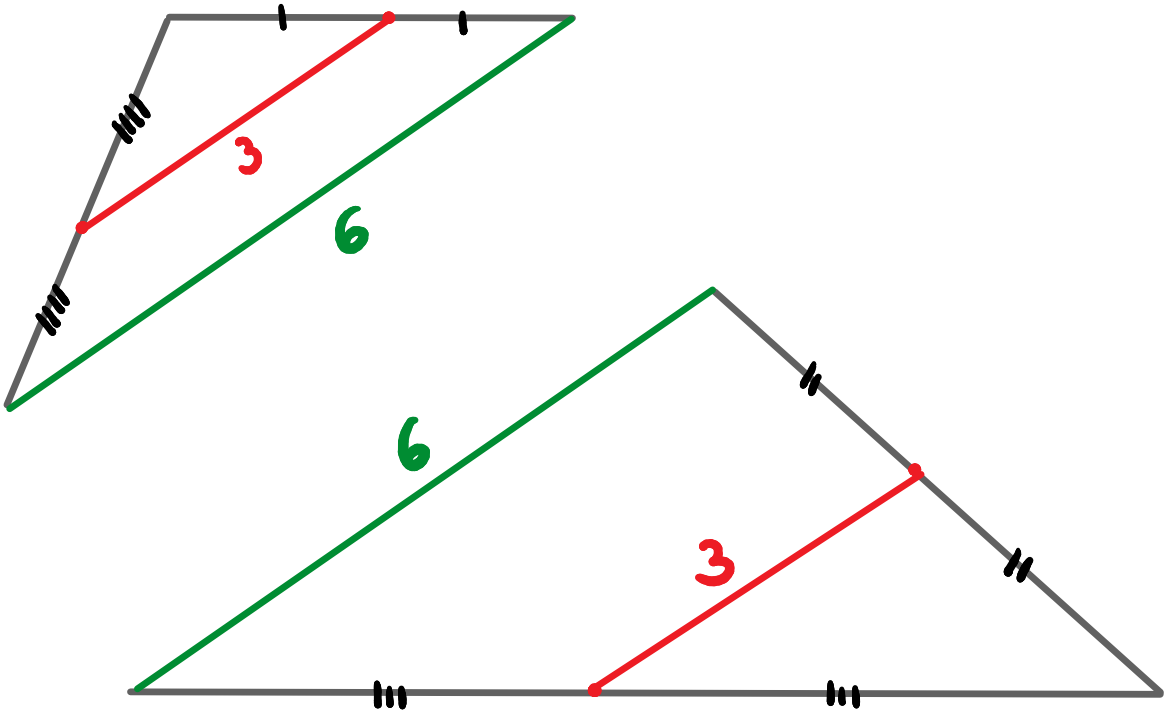
EXEMPLO

AS DIAGONAIS DE UM TRAPÉZIO MEDEM 6 E 8.

CALCULE O PERÍMETRO DO QUADRILÁTERO CUJOS VÉRTICES SÃO OS PONTOS MÉDIOS DOS LADOS DESSE TRAPÉZIO.



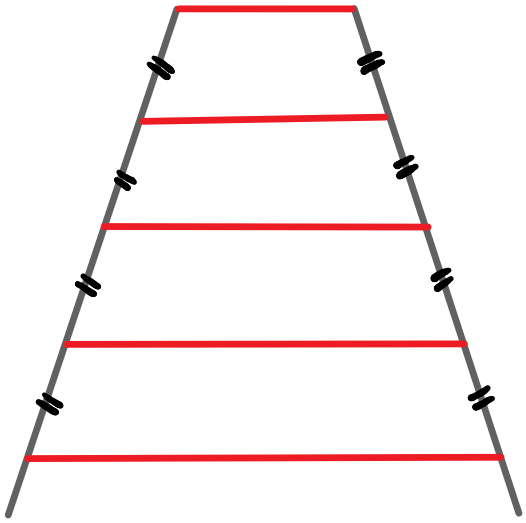
PER. = 4 + 4 + 3 + 3 = 14

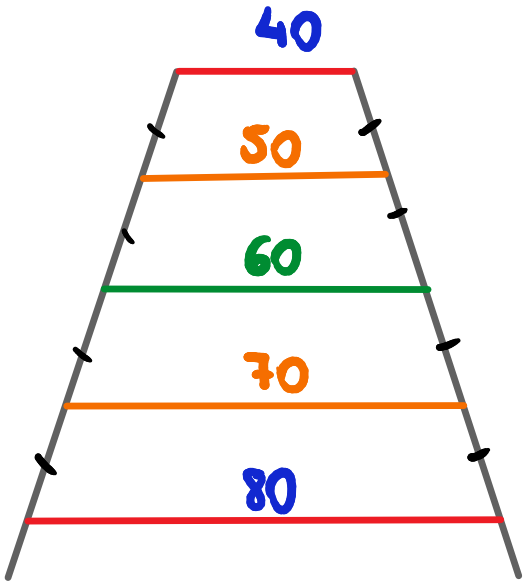


EXEMPLO

DESEJA-SE CONSTRUIR UMA ESCADA COM FORMATO TRAPEZOIDAL DE 5 DEGRAUS, COMO MOSTRA A FIGURA.

SE O MAIOR E O MENOR DEGRAU MEDEM RESPECTIVAMENTE 80cm E 40cm, DETERMINE O COMPRIMENTO DE UMA BARRA DE MADEIRA A PARTIR DA QUAL PODE-SE FABRICAR TODOS OS DEGRAUS.





$$x = 40 + 50 + 60 + 70 + 80$$

$$x = 300 \text{ cm}$$

EXEMPLO

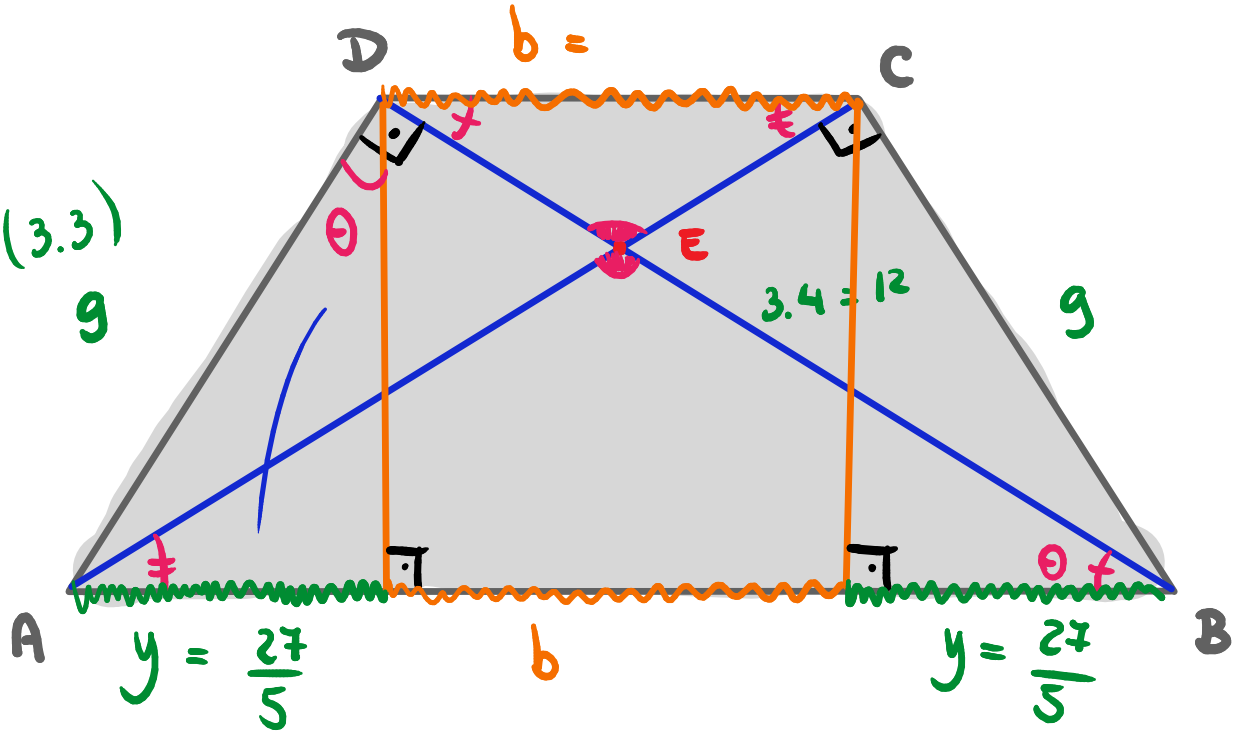
SEJA ABCD UM TRAPÉZIO ISÓCELES COM BASE MAIOR AB. SABE-SE QUE:

$AB = 15$;

$AD = 9$;

$A\hat{D}B = 90°$.

CALCULE A DISTÂNCIA DO PONTO E, INTERSEÇÃO DAS DIAGONAIS, AO LADO AB.



15 (3.5)

$\text{Sen}\theta = \frac{9}{15} = \frac{3}{5}$; $\cos\theta = \frac{4}{5}$

$\text{Sen}\theta = \frac{y}{9}$

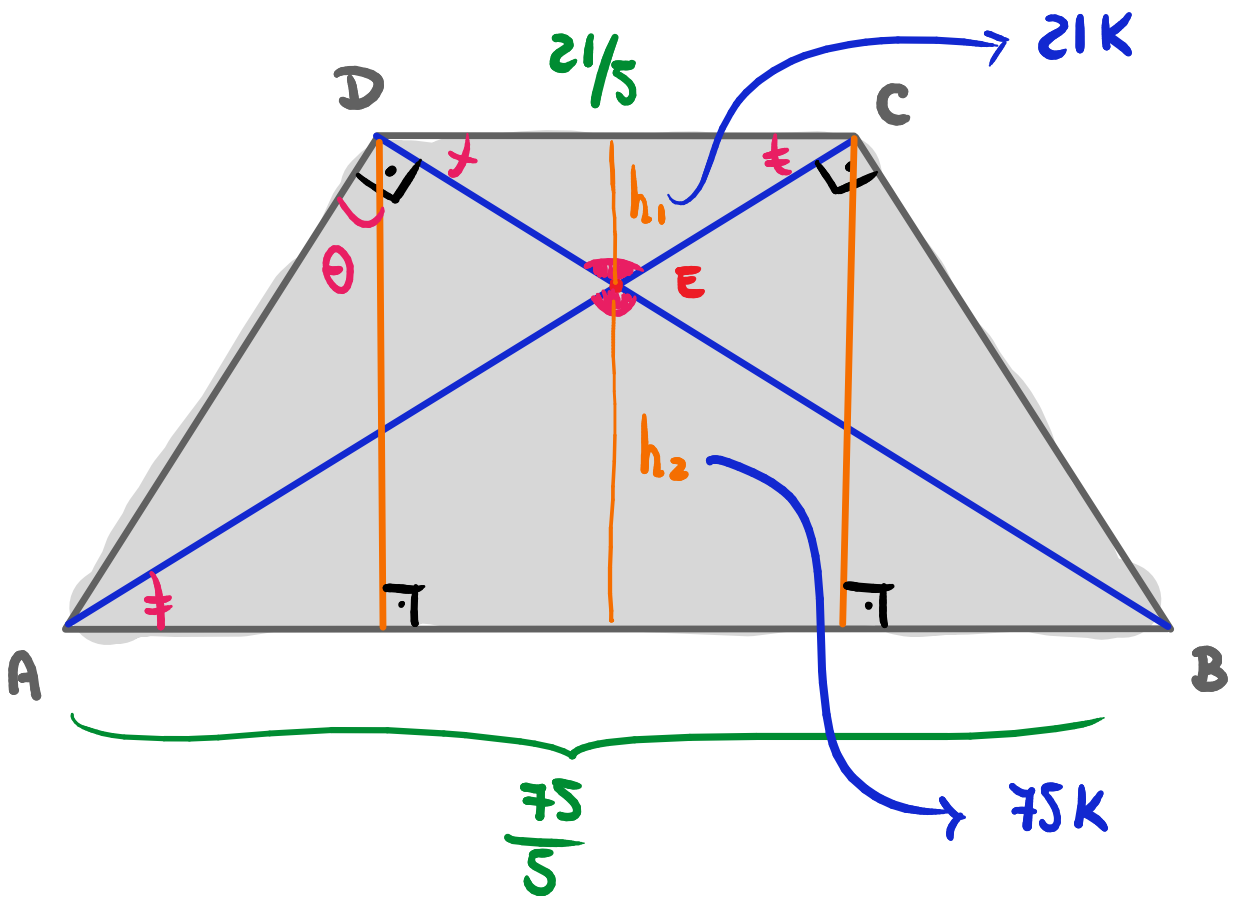
$\frac{3}{5} = \frac{y}{9}$

$y = \frac{27}{5}$

$b = 15 - 2.\frac{27}{5}$

$b = \frac{75 - 54}{5}$

$\boxed{b = \frac{21}{5}}$



$$\cos\theta = \frac{h}{9} \rightarrow \frac{4}{5} = \frac{h}{9} \rightarrow h = \frac{36}{5}$$

$$h_1 + h_2 = h$$

$$21K + 75K = \frac{36}{5}$$

$$96K = \frac{36}{5}$$

$$K = \frac{3}{40}$$

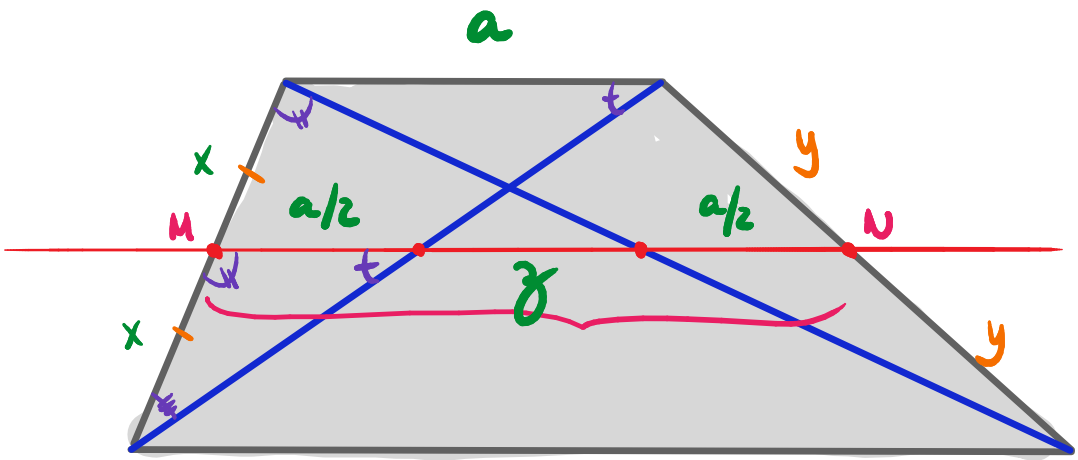
$$h_2 = \frac{75}{1} \cdot \frac{3}{40}$$

$$h = \frac{45}{8}$$

EXEMPLO

SEJA UM TRAPÉZIO DE BASES a E b.

CALCULE O COMPRIMENTO DO SEGMENTO QUE LIGA OS PONTOS MÉDIOS DAS DIAGONAIS DESSE TRAPÉZIO.



$$MN = \frac{a + b}{2}$$

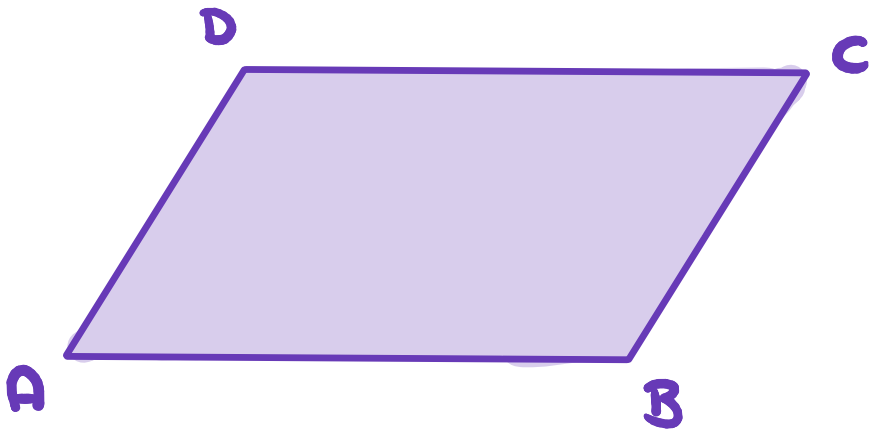
$$\gamma = \frac{a + b}{2} - \frac{a}{2} - \frac{a}{2}$$

$$\boxed{\gamma = \frac{b - a}{2}}$$

→ MEDIANA DE EULER

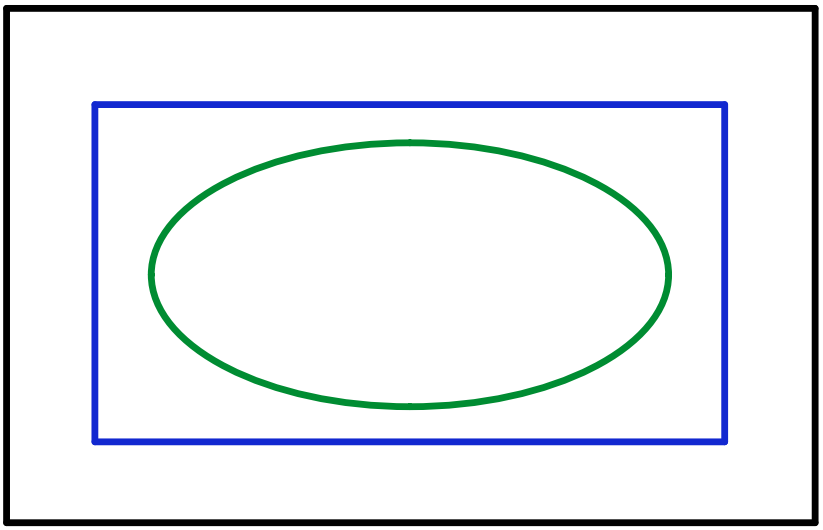
PARALELOGRAMO

PARALELOGRAMOS SÃO QUADRILÁTEROS QUE POSSUEM LADOS OPOSTOS PARALELOS.



$AB \parallel CD$

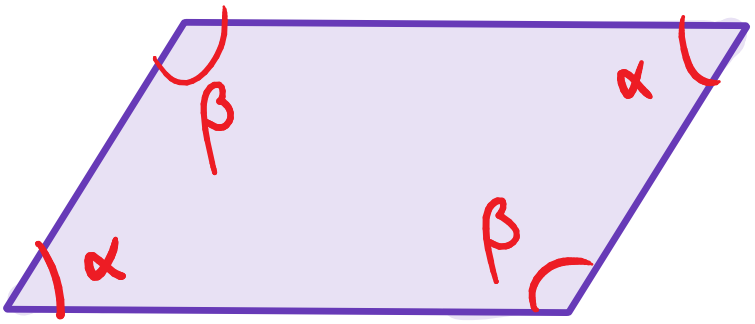
$BC \parallel AD$



- QUADRILÁTEROS
- TRAPÉZIOS
- PARALELOGRAMOS

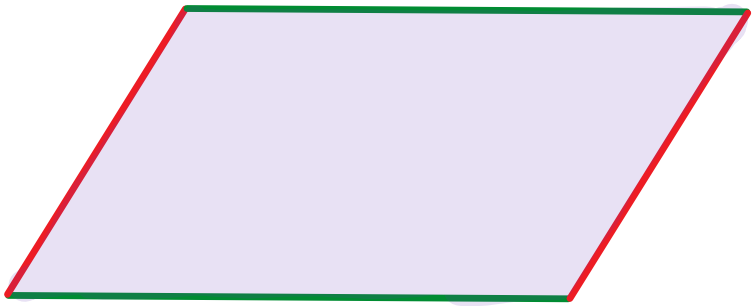
PROPRIEDADES:

I- ÂNGULOS

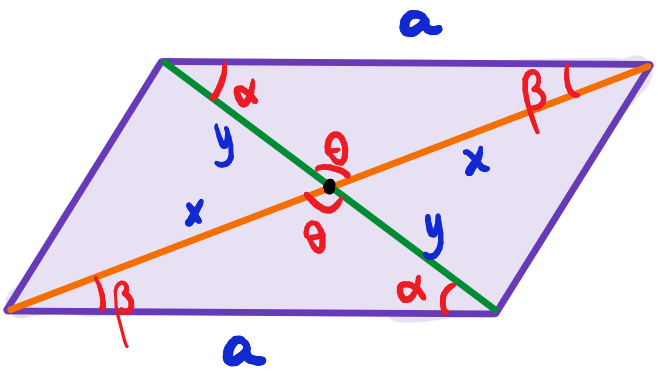


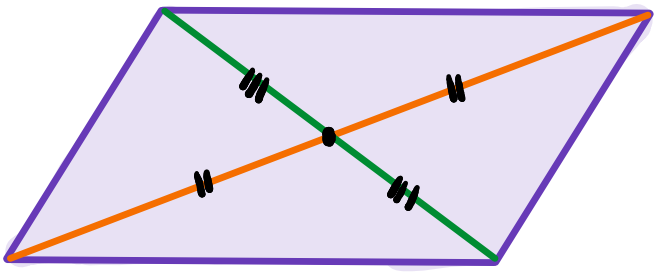
$$\alpha + \beta = 180°$$

II- LADOS OPOSTOS SÃO CONGRUENTES



III- DIAGONAIS SE CORTAM EM SEUS PONTOS MÉDIOS.

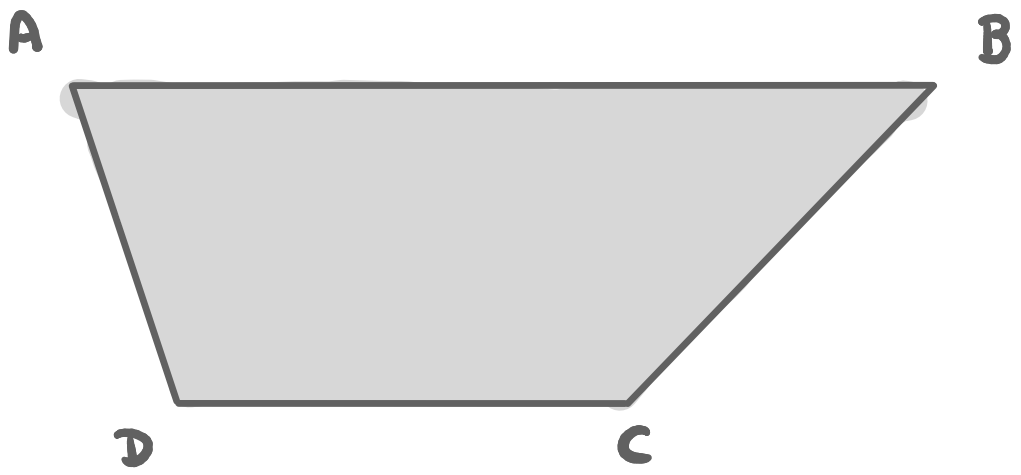


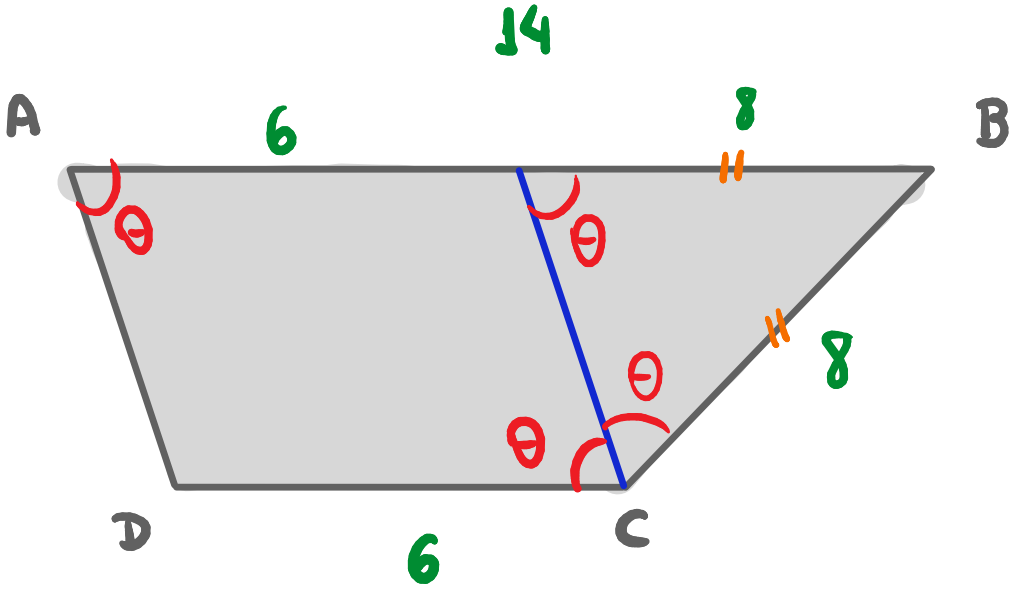


EXEMPLO

NO TRAPÉZIO ABAIXO, AB = 14 E CD = 6. ALÉM DISSO, O ÂNGULO C É O DOBRO DO ÂNGULO A.

DETERMINE O COMPRIMENTO DO LADO BC.





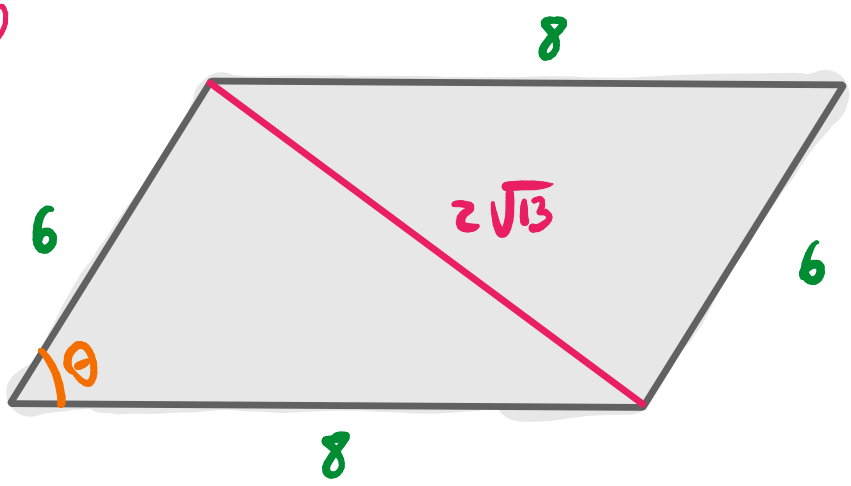
$BC = 8$

EXEMPLO

OS LADOS DE UM PARALELOGRAMO MEDEM 6 E 8, E UMA DE SUAS DIAGONAIS MEDE $2\sqrt{13}$.

DETERMINE O COMPRIMENTO DA OUTRA DIAGONAL.

$2\sqrt{13} < 10$

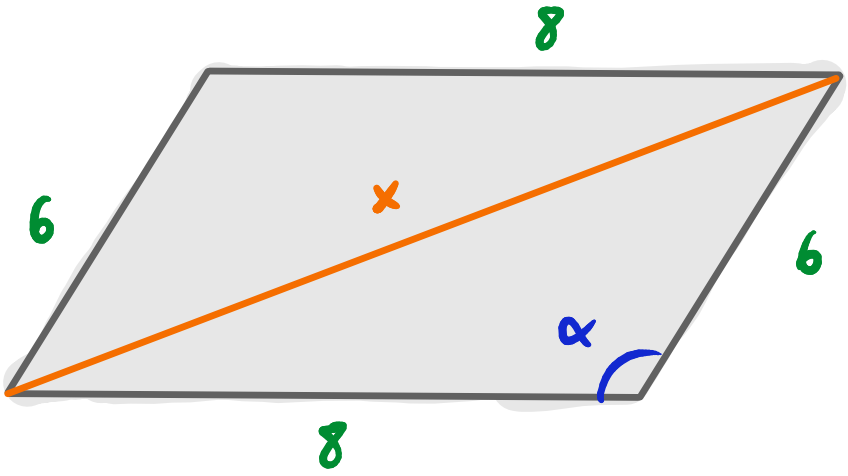


$$(2\sqrt{13})^2 = 6^2 + 8^2 - 2.6.8.\cos\theta$$

$$52 = 36 + 64 - 2.6.8\cos\theta$$

$$2.6.8.\cos\theta = 48$$

$$\boxed{\cos\theta = \frac{1}{2}}$$



$$\cos \alpha = -\cos\theta = -\frac{1}{2}$$

$$x^2 = 6^2 + 8^2 - 2 \cdot 6 \cdot 8 \cdot \left(\frac{-1}{2}\right)$$

$$x^2 = 100 + 48$$

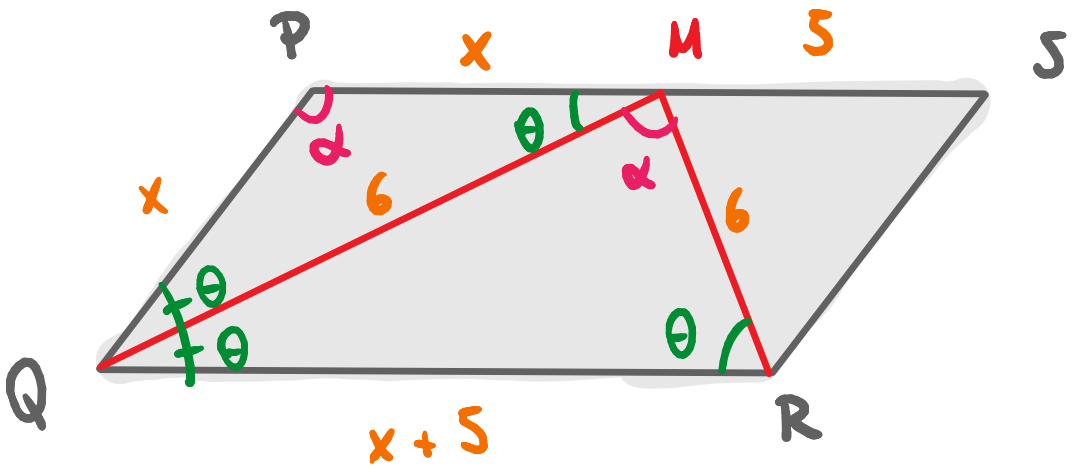
$$x = \sqrt{148}$$

$$x = 2\sqrt{37}$$

EXEMPLO

O PARALELOGRAMO PQRS É TAL QUE A BISSETRIZ DO ÂNGULO AGUDO Q INTERSECTA O LADO PS NO PONTO M COM MS = 5 E MQ = MR = 6.

DETERMINE O COMPRIMENTO DO LADO PQ.



$$\Delta PQM \sim \Delta MQS$$

$$\frac{x}{6} = \frac{6}{x+5}$$

$$x^2 + 5x = 36$$

$$x + 5x - 36 = 0$$

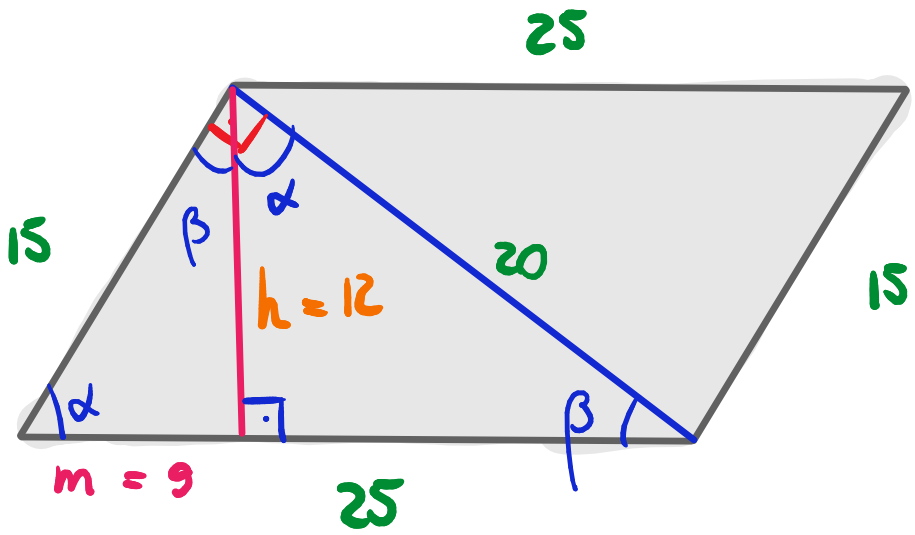
$$S = \frac{-b}{a} = -5 \quad ; \quad P = \frac{c}{a} = -36$$

$$x = 4 \text{ ou } x = -9$$

$$\underline{PQ = 4}$$

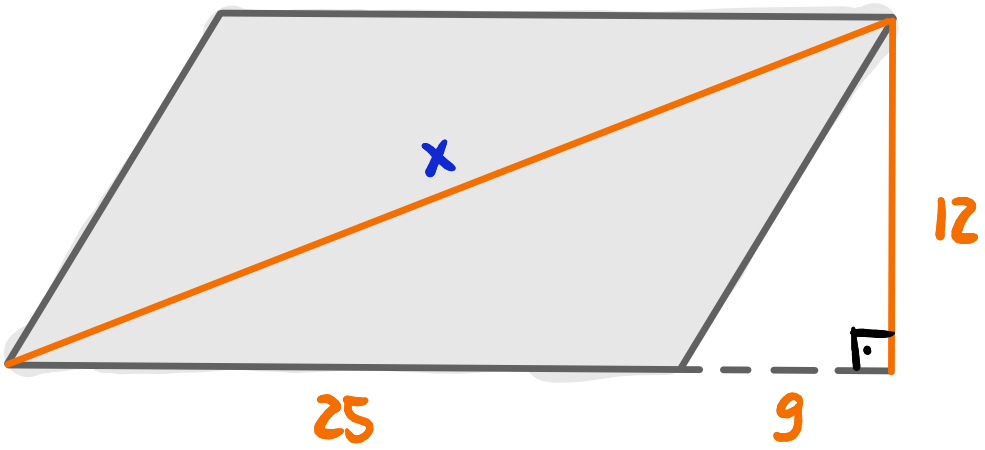
EXEMPLO

DOIS DOS LADOS DE UM PARALELOGRAMO MEDEM 15 E 25. SE UMA DAS DIAGONAIS FORMA 90° COM UM DOS LADOS, DETERMINE O COMPRIMENTO DA MAIOR DIAGONAL DESSE PARALELOGRAMO.



$$\frac{m}{15} = \frac{15}{25} \rightarrow \frac{\cancel{5}^{3}}{\cancel{25}}\, m = \frac{\overset{3}{\cancel{15}} \cdot \overset{3}{\cancel{15}}}{\cancel{5}}$$

$$m = 9$$



$$x^2 = 34^2 + 12^2$$

$$x^2 = 1156 + 144$$

$$x^2 = 1300$$

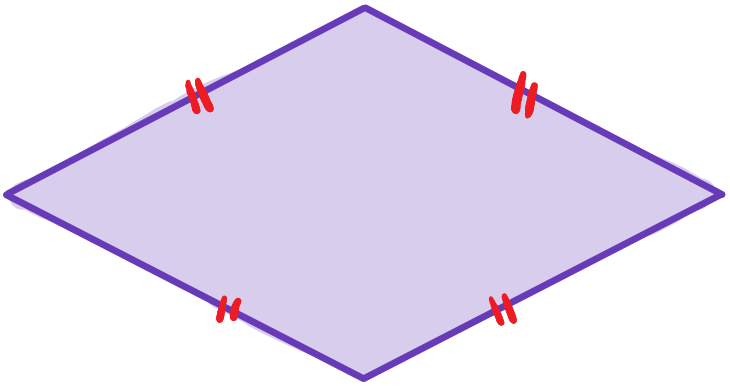
$$x = 10\sqrt{13}$$

34 × 34

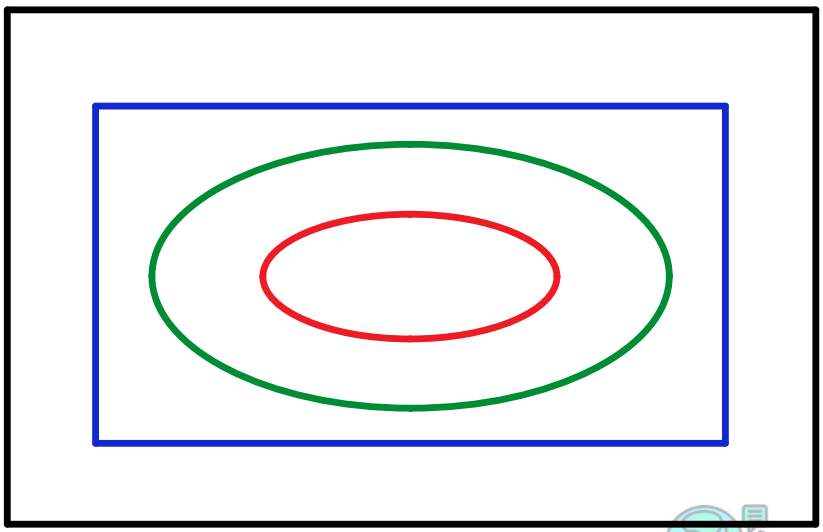
13 6
10 2
1156

LOSANGO

LOSANGOS SÃO QUADRILÁTEROS CONVEXOS QUE POSSUEM OS LADOS CONGRUENTES.



LOSANGOS SÃO PARALELOGRAMOS.



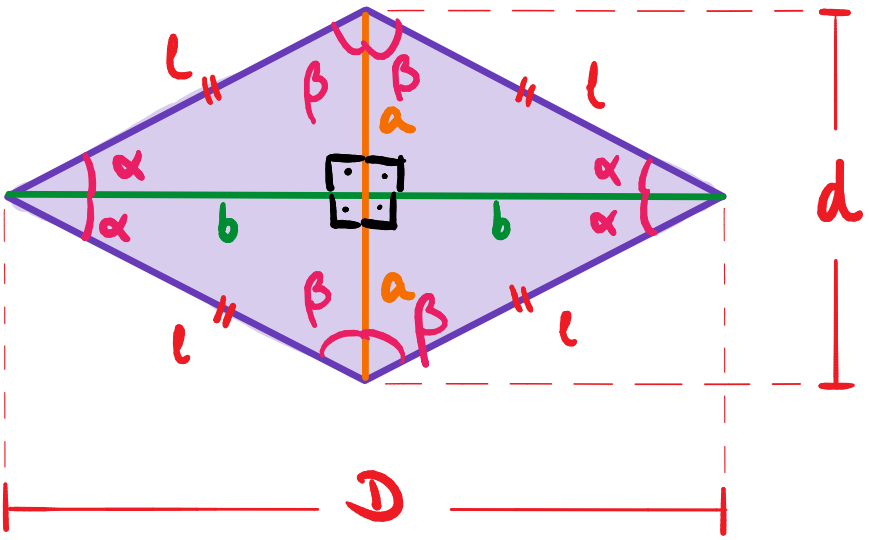
- QUADRILÁTEROS
- TRAPÉZIOS
- PARALELOGRAMOS
- LOSANGOS

PROPRIEDADES:

I- TODAS AS PROPRIEDADES DO PARALELOGRAMO.

- LADOS OPOSTOS IGUAIS.
- ÂNGULOS OPOSTOS IGUAIS.
- ÂNGULOS CONSECUTIVOS SUPLEMENTARES.
- DIAGONAIS SE INTERSEPTAM NOS SEUS PONTOS MÉDIOS

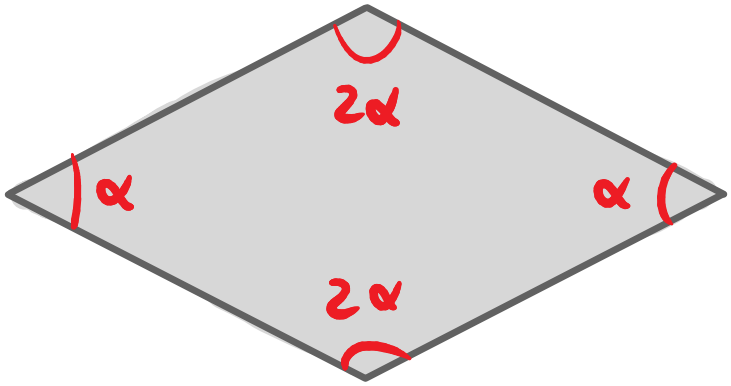
II- DIAGONAIS SE INTERSEPTAM FORMANDO 90° E SÃO BISSETRIZES DOS ÂNGULOS.



EXEMPLO

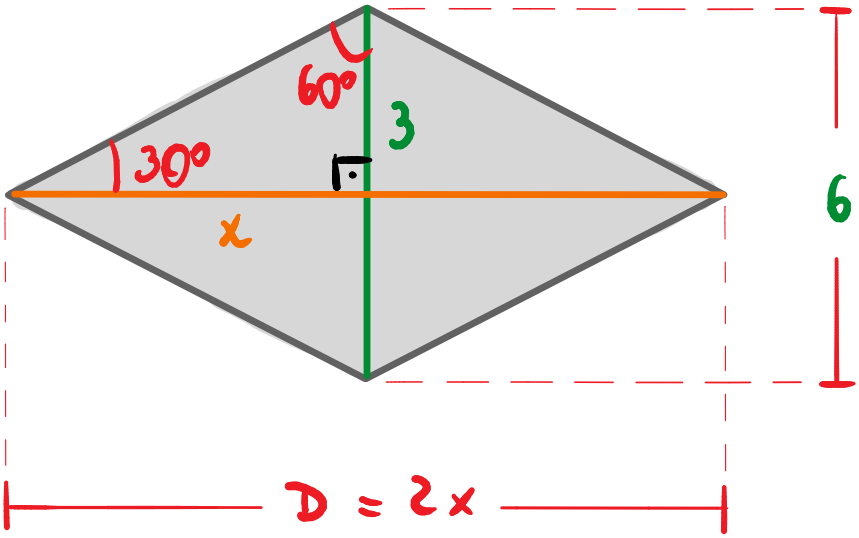
O MENOR ÂNGULO DE UM LOSANGO É METADE DO MAIOR ÂNGULO DESSE LOSANGO.

SE A MENOR DIAGONAL MEDE 6, CALCULE A MEDIDA DA MAIOR DIAGONAL DESSE LOSANGO.



$$\alpha + 2\alpha = 180^\circ$$

$$\alpha = 60^\circ$$



$$tg\,60^\circ = \frac{x}{3}$$

$$\sqrt{3} = \frac{x}{3}$$

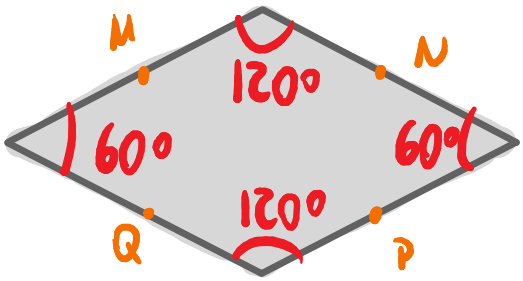
$$x = 3\sqrt{3} \rightarrow D = 2 \cdot 3\sqrt{3}$$

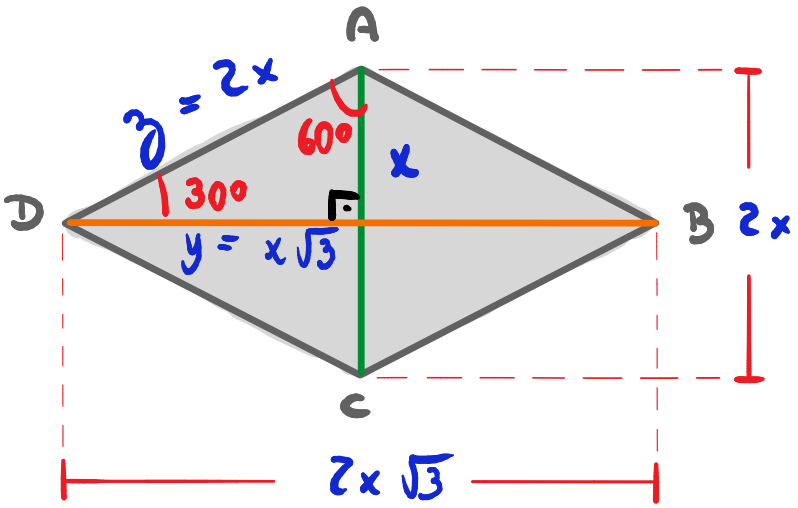
$$D = 6\sqrt{3}$$

EXEMPLO

SEJAM M, N, P E Q OS PONTOS MÉDIOS DOS LADOS DO LOSANGO ABCD.

SE UM DOS ÂNGULOS DESSE LOSANGO MEDE 60°, CALCULE A RAZÃO ENTRE O PERÍMETRO DO QUADRILÁTERO ABCD E DO QUADRILÁTERO MNPQ.





$$tg\,60° = \frac{y}{x}$$

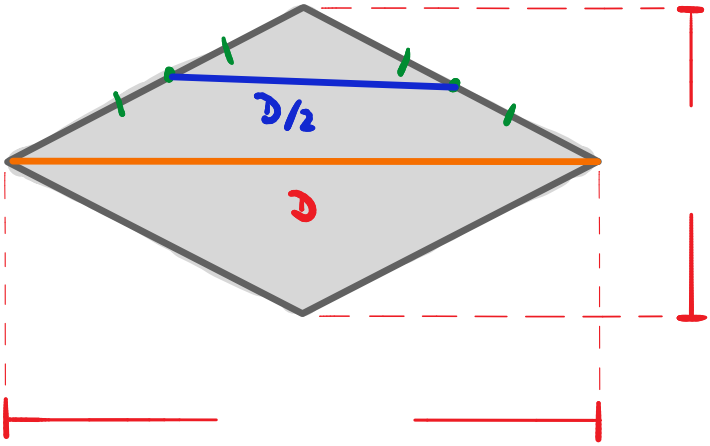
$$\sqrt{3} = \frac{y}{x}$$

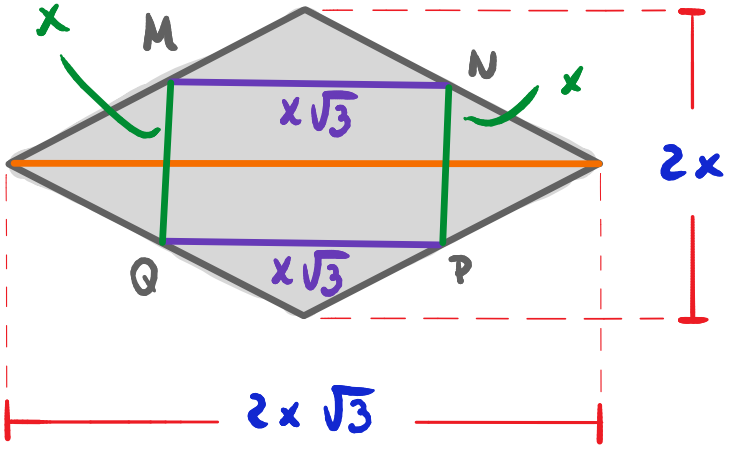
$$y = x\sqrt{3}$$

$$Sen\,30° = \frac{x}{\partial} \rightarrow \frac{1}{2} = \frac{x}{\partial} \rightarrow \partial = 2x$$

$$PER(ABCD) = 4 \cdot 2x$$

$$PER(ABCD) = 8x$$





$$\text{PER}(MNPQ) = 2x\sqrt{3} + 2x$$

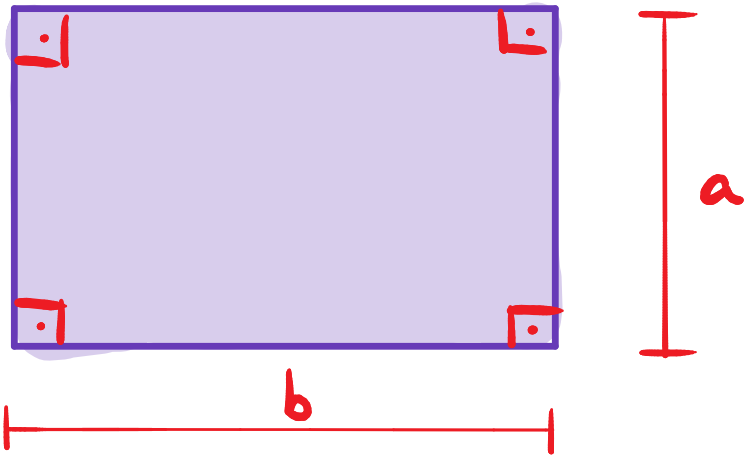
$$\text{PER}(MNPQ) = 2x(\sqrt{3}+1)$$

$$\frac{\text{PER}(ABCD)}{\text{PER}(MNPQ)} = \frac{8\cancel{x}}{\cancel{2x}(\sqrt{3}+1)} = \frac{4}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

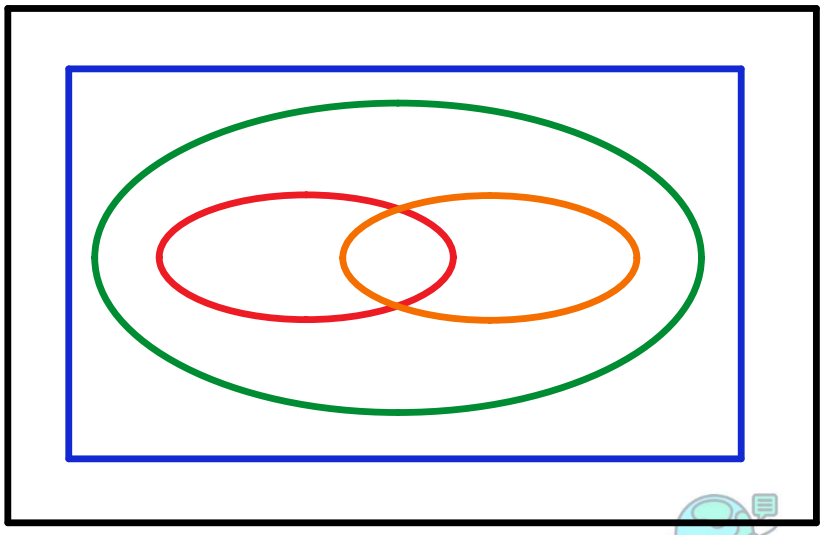
$$= \frac{4(\sqrt{3}-1)}{3-1} = 2(\sqrt{3}-1)$$

RETÂNGULO

RETÂNGULOS SÃO QUADRILÁTEROS QUE POSSUEM OS QUATRO ÂNGULOS CONGRUENTES (NO CASO, 90°)



RETÂNGULOS SÃO PARALELOGRAMOS.



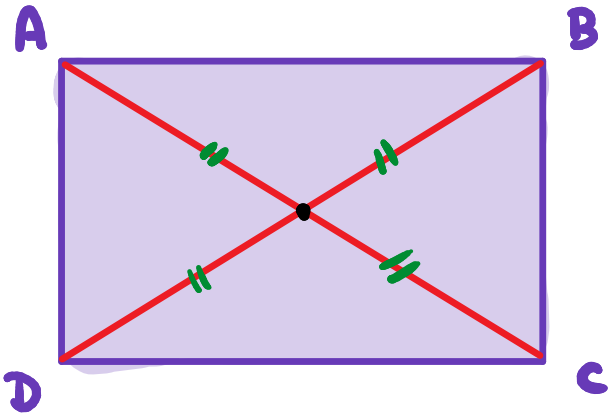
- QUADRILÁTEROS
- TRAPÉZIOS
- PARALELOGRAMOS
- LOSANGOS
- RETÂNGULO

PROPRIEDADES:

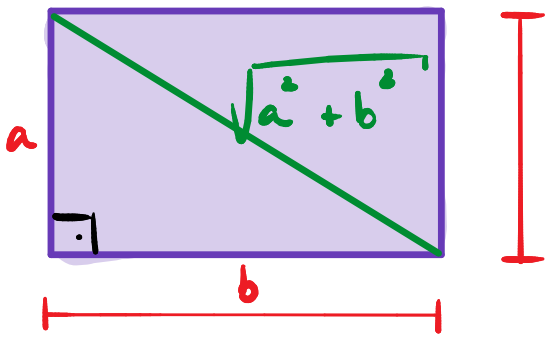
I- TODAS AS PROPRIEDADES DO PARALELOGRAMO.

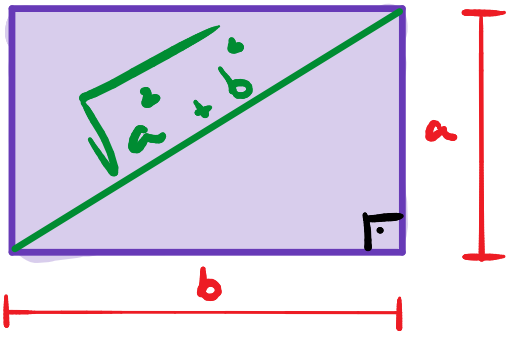
- LADOS OPOSTOS IGUAIS.
- ÂNGULOS OPOSTOS IGUAIS.
- ÂNGULOS CONSECUTIVOS SUPLEMENTARES.
- DIAGONAIS SE INTERSEPTAM NOS SEUS PONTOS MÉDIOS

II- AS DIAGONAIS SÃO CONGRUENTES.



AC = BD

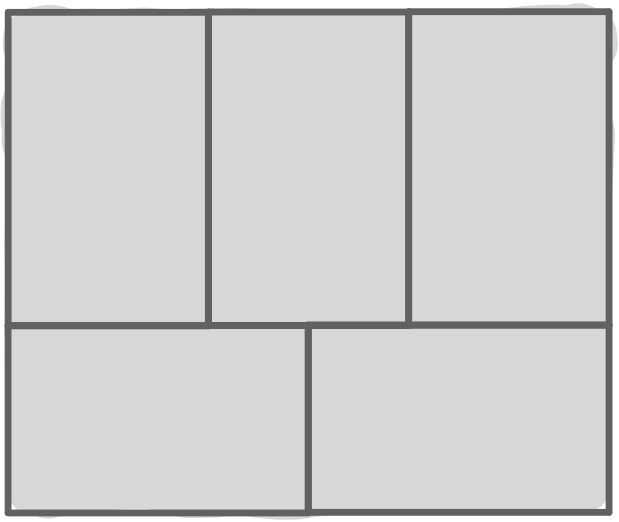


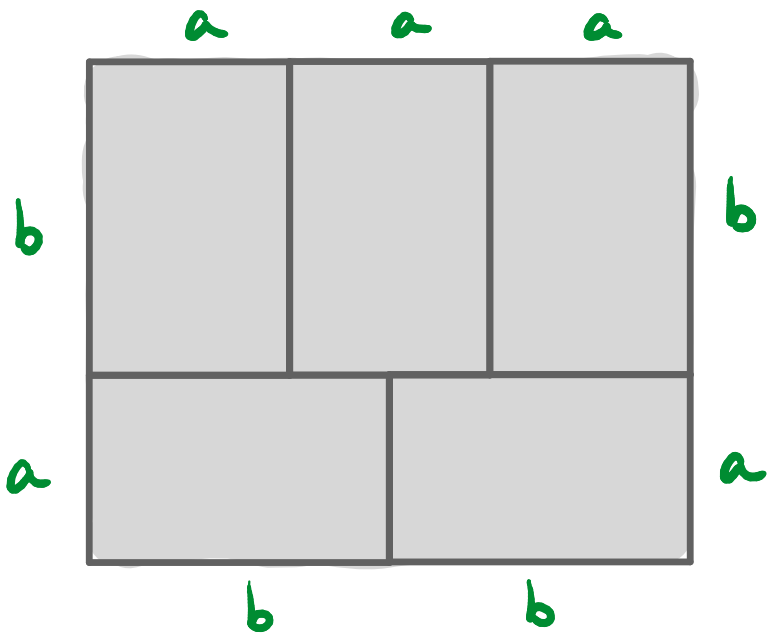


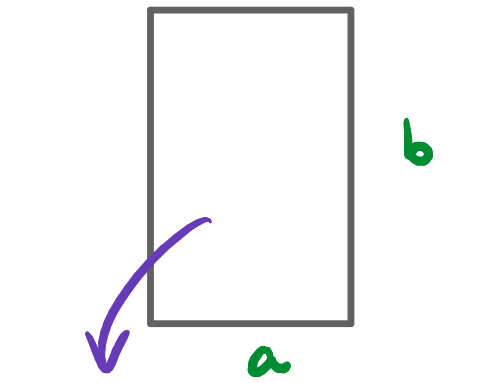
EXEMPLO

O TERRENO MOSTRADO NA FIGURA POSSUI 176 DE PERÍMETRO E É FORMADO POR 5 RETÂNGULOS CONGRUENTES.

DETERMINE O PERÍMETRO DE CADA RETÂNGULO.







$P_{ER} = 2(a + b)$

$5a + 4b = 176$

$3a = 2b \rightarrow 6a = 4b$

$5a + 6a = 176$

$11a = 176$

$a = 16$

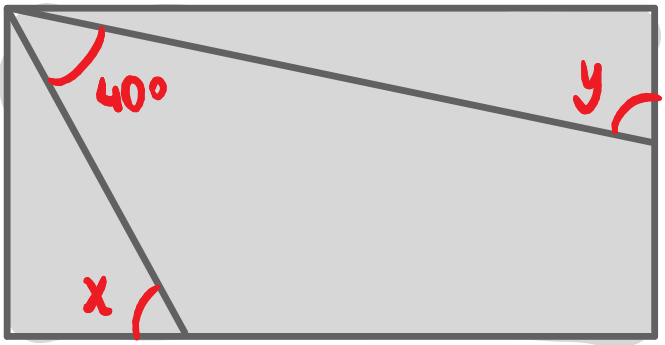
$3.16 = 2.b$

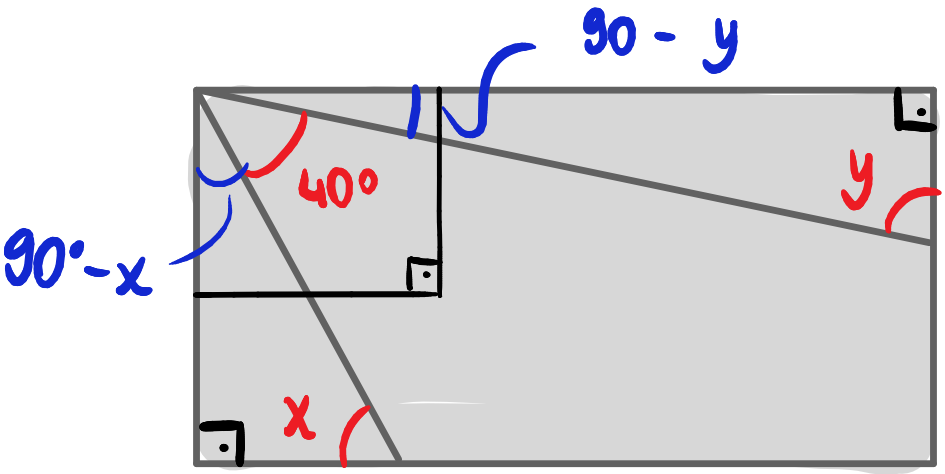
$b = 24$

$P_{ER} = 2(16 + 24) \rightarrow P_{ER} = 80$

EXEMPLO

CALCULE O VALOR DA SOMA, EM GRAUS, DOS ÂNGULOS x E y NO RETÂNGULO ABAIXO.





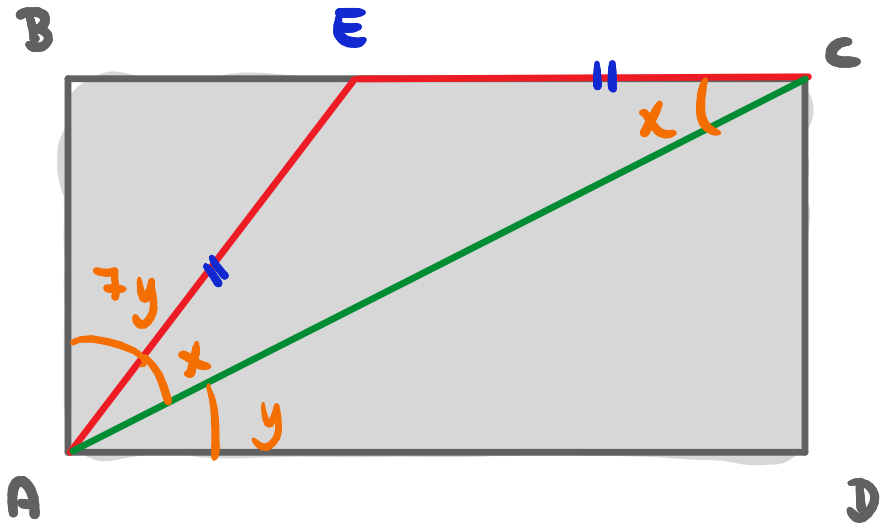
$$40 + 90 - x + \cancel{90} - y = \cancel{90}$$

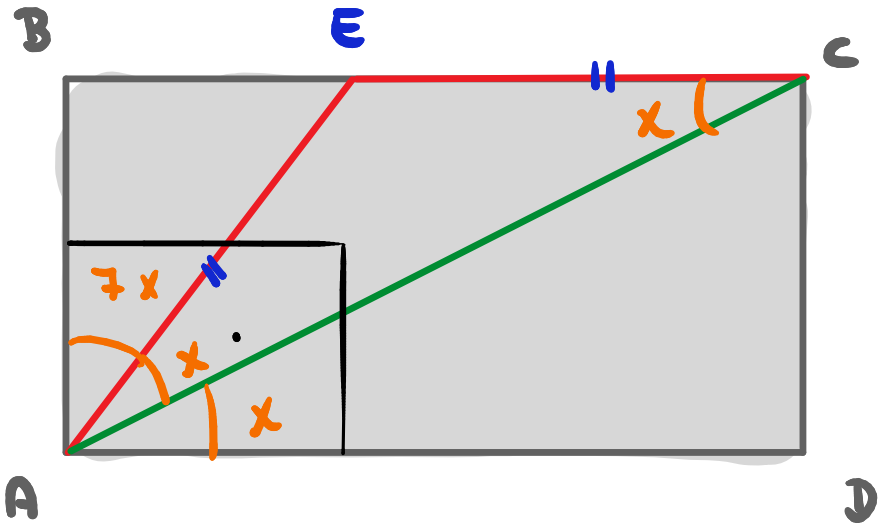
$$130° = x + y$$

EXEMPLO

NO RETÂNGULO ABCD, E É UM PONTO DO LADO BC EQUIDISTANTE DE A E C.

CALCULE O VALOR DO ÂNGULO EÂC SABENDO QUE A MEDIDA DO ÂNGULO BÂE É SETE VEZES A MEDIDA DO ÂNGULO CÂD.





$$9x = 90°$$

$$x = 10°$$

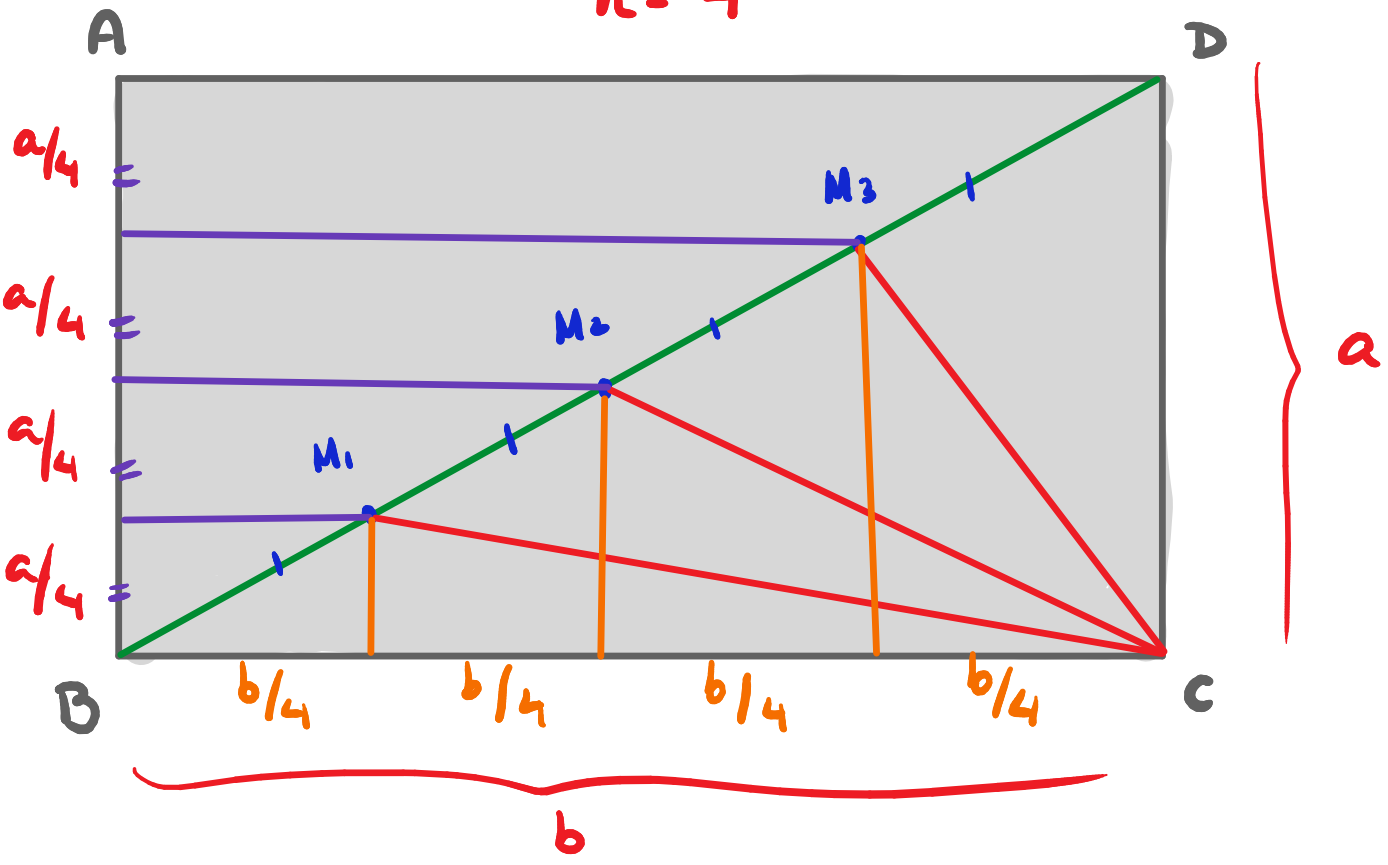
$$E\hat{A}C = 10°$$

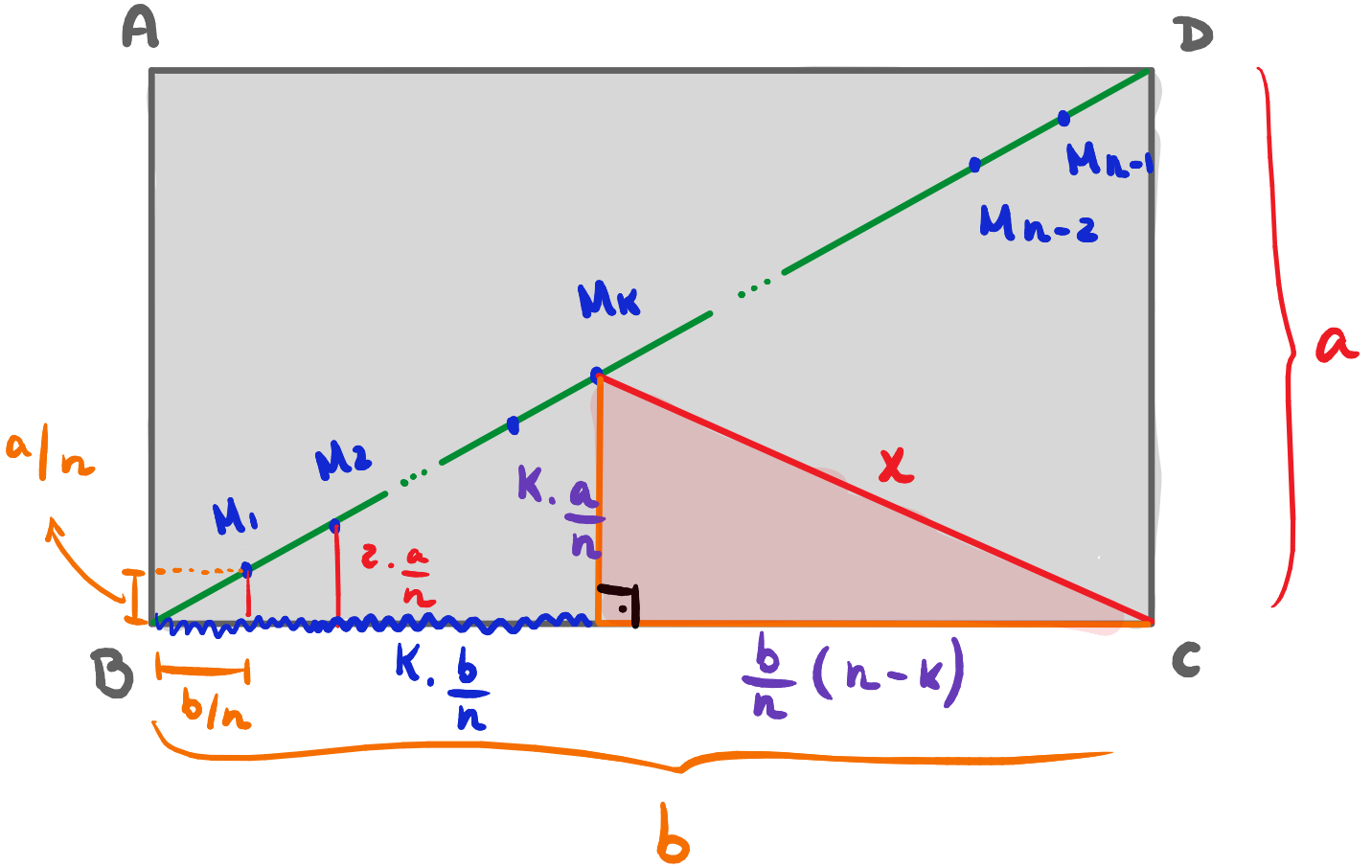
EXEMPLO

DADO UM RETÂNGULO ABCD, DE LADOS a E b, DIVIDE-SE A DIAGONAL $\overline{BD}$ EM n SEGMENTOS IGUAIS, MARCANDO-SE, A PARTIR DE B, OS PONTOS M_1, M_2, ... , M_{n-1}.

DETERMINE UMA EXPRESSÃO PARA A MEDIDA DO SEGMENTO $\overline{CM_K}$, EM FUNÇÃO DE a, b, n E K.

$n = 4$





$$b - \frac{Kb}{n} = b\left(\frac{n-K}{n}\right) = \frac{b}{n}(n-k)$$

$$x = CM_K = \sqrt{\left(K \cdot \frac{a}{n}\right)^2 + \left(\frac{b}{n}(n-K)\right)^2}$$

$$x = \sqrt{\frac{K^2 \cdot a^2}{n^2} + \frac{b^2}{n^2}(n-K)^2}$$

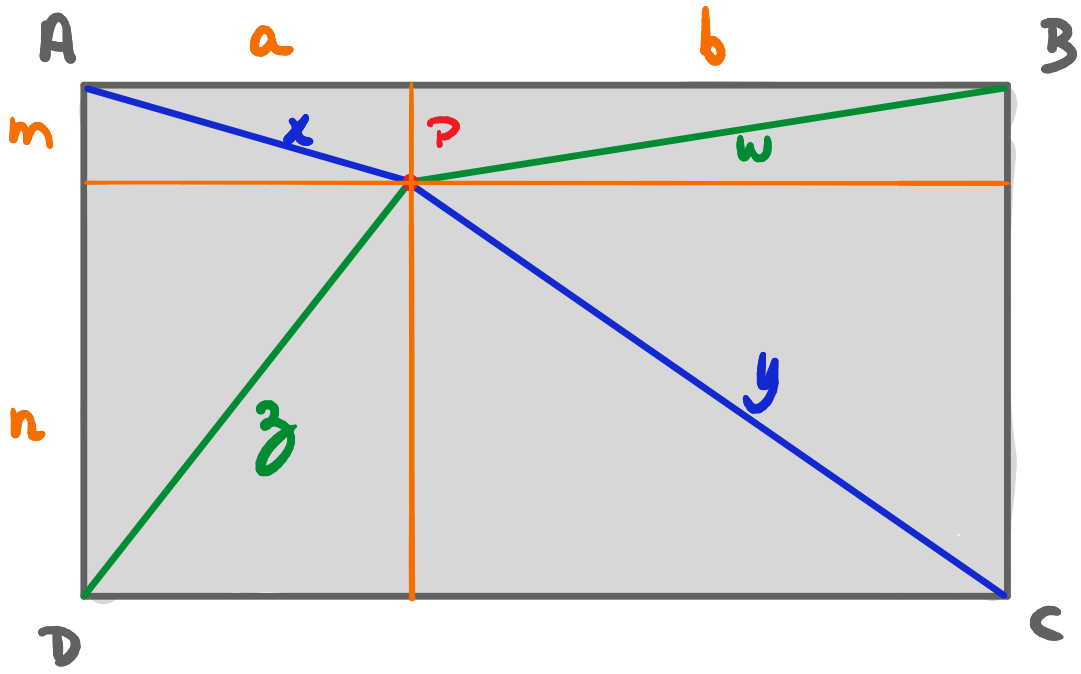
$$x = \sqrt{\frac{1}{n^2}\left[(Ka)^2 + b^2(n-K)^2\right]}$$

$$\boxed{x = \frac{1}{n}\sqrt{a^2K^2 + b^2(n-K)^2}}$$

EXEMPLO

SEJA O RETÂNGULO ABCD E UM PONTO P.

SE $PA^2 + PC^2 = 27$, CALCULE O VALOR DE $PB^2 + PD^2$



$$\begin{cases} x^2 = a^2 + m^2 \\ y^2 = b^2 + n^2 \end{cases}$$

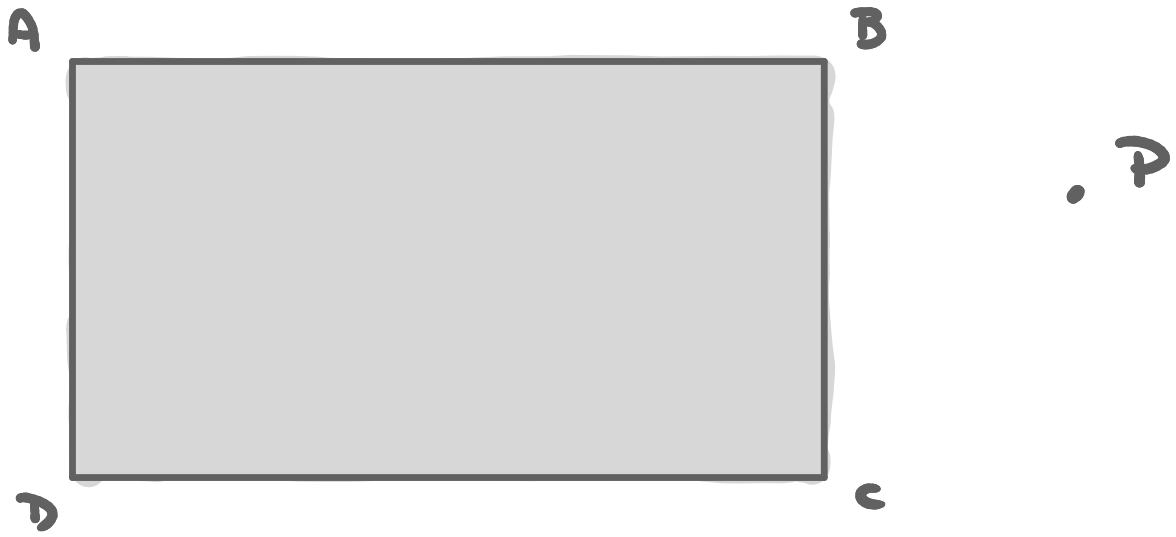
$$\rightarrow \quad x^2 + y^2 = a^2 + b^2 + m^2 + n^2$$

$$\begin{cases} w^2 = b^2 + m^2 \\ z^2 = a^2 + n^2 \end{cases}$$

$$\rightarrow \quad w^2 + z^2 = a^2 + b^2 + m^2 + n^2$$

$$\boxed{PA^2 + PC^2 = PB^2 + PC^2}$$

TEOREMA DE MARLEN



$$PA^2 + PC^2 = PB^2 + PD^2$$

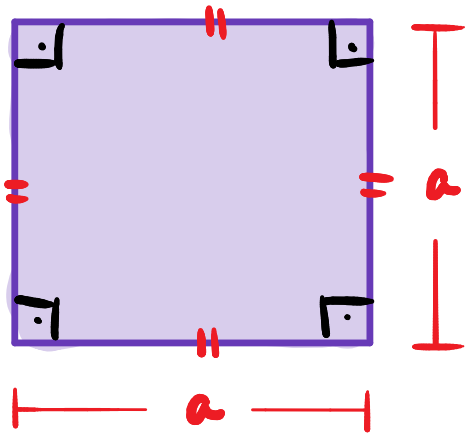
PONTO P QUALQUER!

INCLUSIVE FORA DO PLANO

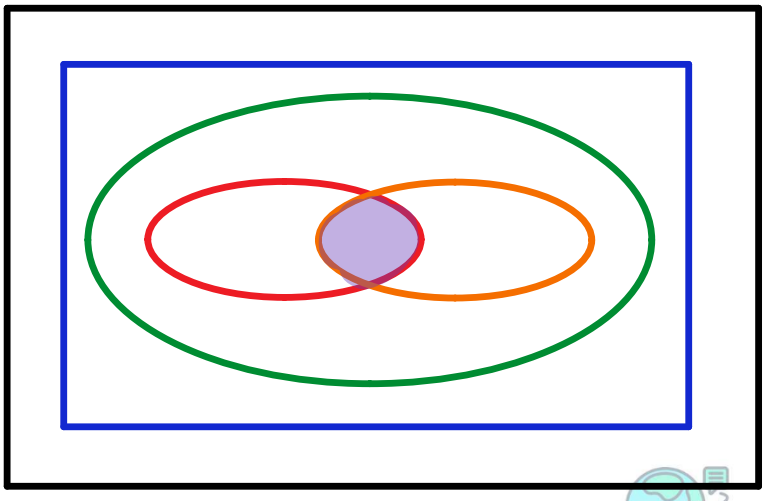
$PA^2 + PC^2 = 27 \rightarrow PB^2 + PD^2 = 27$

QUADRADO

QUADRADOS SÃO QUADRILÁTEROS QUE POSSUEM OS QUATRO ÂNGULOS CONGRUENTES E OS QUADRO LADOS TAMBÉM CONGRUENTES.



QUADRADOS SÃO LOSANGOS E RETÂNGULOS.



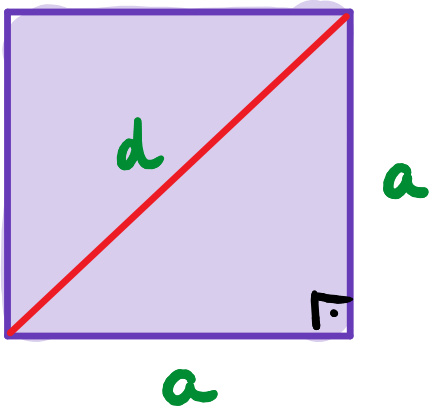
- QUADRILÁTEROS
- TRAPÉZIOS
- PARALELOGRAMOS
- LOSANGOS
- RETÂNGULO
- QUADRADO

PROPRIEDADES:

I- TODAS AS PROPRIEDADES DOS LOSANGOS E RETÂNGULOS.

- LADOS OPOSTOS IGUAIS.
- ÂNGULOS OPOSTOS IGUAIS.
- ÂNGULOS CONSECUTIVOS SUPLEMENTARES.
- DIAGONAIS SE INTERSEPTAM NOS SEUS PONTOS MÉDIOS
- DIAGONAIS FORMANDO 90°.
- DIAGONAIS CONGRUENTES.

II- COMPRIMENTO DA DIAGONAL:



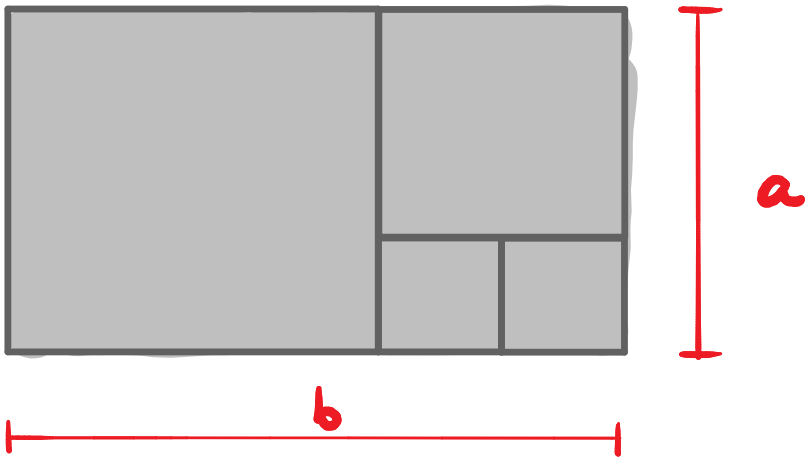
$$d^2 = a^2 + a^2$$

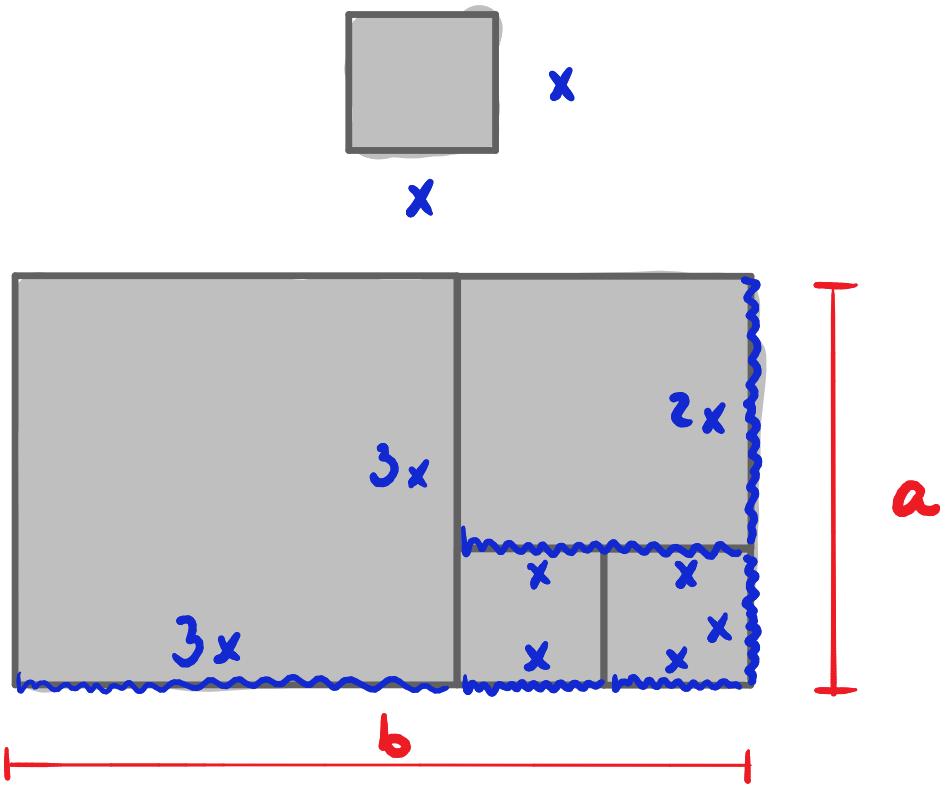
$$d^2 = 2a^2$$

$$\boxed{d = a\sqrt{2}}$$

EXEMPLO

O RETÂNGULO ABAIXO DE DIMENSÕES a E b FOI SUBDIVIDIDO EM QUADRADOS. DETERMINE O VALOR DA RAZÃO a/b.





$a = 3x$ $\qquad$ $b = 5x$

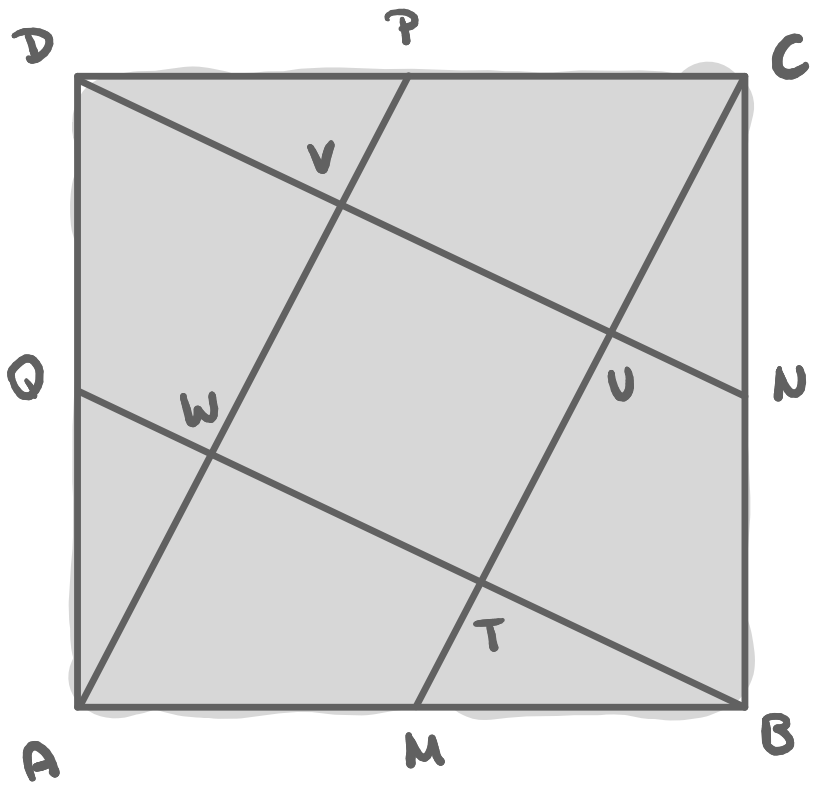
$$\frac{a}{b} = \frac{3x}{5x}$$

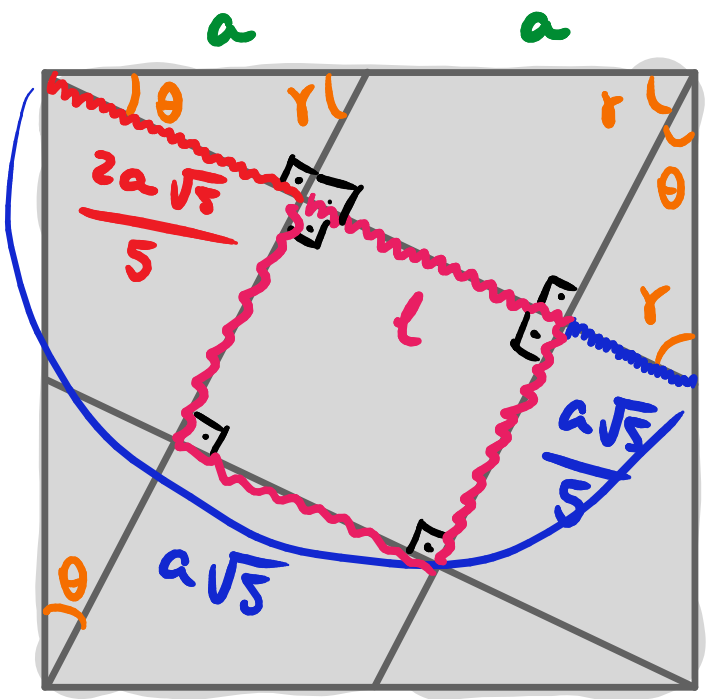
$$\frac{a}{b} = \frac{3}{5}$$

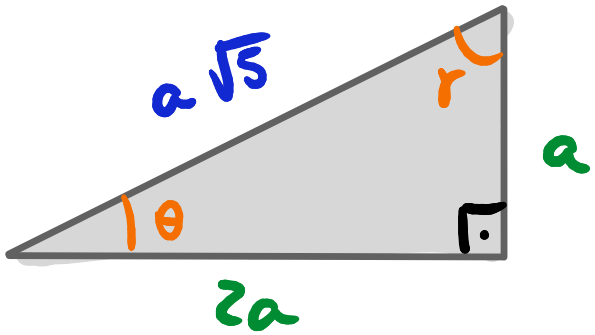
EXEMPLO

SEJA O QUADRADO ABCD DE LADO 5.

SE OS PONTOS M, N, P e Q SÃO PONTOS MÉDIOS DOS LADOS DESSE QUADRADO, DETERMINE A MEDIDA DO LADO DO QUADRILÁTERO TUVW.

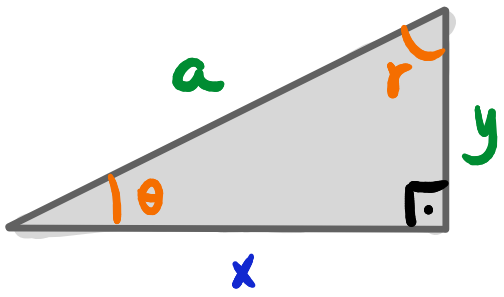






$$\cos\theta = \frac{2a}{a\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\operatorname{sen}\theta = \frac{a}{a\sqrt{5}} = \frac{\sqrt{5}}{5}$$



$$\cos\theta = \frac{x}{a}$$

$$\frac{2\sqrt{5}}{5} = \frac{x}{a} \rightarrow x = \frac{2a\sqrt{5}}{5}$$

$$\operatorname{sen}\theta = \frac{y}{a}$$

$$\frac{\sqrt{5}}{5} = \frac{y}{a} \rightarrow y = \frac{a\sqrt{5}}{5}$$

$$l = a\sqrt{5} - \frac{a\sqrt{5}}{5} - \frac{2a\sqrt{5}}{5}$$

$$l = \frac{2a\sqrt{5}}{5}$$

$$l = \sqrt{5}$$