

MATRIZES

CONCEITO

MATRIZ É UMA ESTRUTURA NA QUAL NÚMEROS SÃO ORGANIZADOS EM LINHAS E COLUNAS

$$A_{3 \times 2} = \begin{bmatrix} 7 & 4 \\ 5 & 2 \\ -1 & 9 \end{bmatrix}$$

Linha 1
Linha 2
Linha 3

Coluna 1 Coluna 2

$A_{3 \times 2}$: A É UMA MATRIZ DE ORDEM 3×2



REPRESENTAÇÃO GENÉRICA

$$A_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

$$A_{3 \times 2} \rightarrow a_{ij} = \begin{cases} i^2 + j & ; i > j \\ i^2 - j & ; i \leq j \end{cases}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} 1^2 - 1 & 1^2 - 2 \\ 2^2 + 1 & 2^2 - 2 \\ 3^2 + 1 & 3^2 + 2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 5 & 2 \\ 10 & 11 \end{bmatrix}$$



EXEMPLO

CONSIDERANDO A MATRIZ ABAIXO, IDENTIFIQUE:

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 4 & 8 & 7 & 6 \\ 8 & 5 & 1 & -4 \\ 7 & 0 & 9 & 0 \end{bmatrix}$$

ORDEN DA MATRIZ: 4×4

ELEMENTOS $a_{12} = 2$

$$a_{43} = 9$$

$$a_{33} = 1$$

$$a_{14} = 4$$



EXEMPLO

CONSTRUA A MATRIZ DE ORDEM 2×3 DADA

Por :

$$a_{ij} = \begin{cases} 2i + j^2 & ; i \geq j \\ i - j & ; i < j \end{cases}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 \cdot 1 + 1^2 & 1 - 2 & 1 - 3 \\ 2 \cdot 2 + 1^2 & 2 \cdot 2 + 2^2 & 2 - 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 5 & 8 & -1 \end{bmatrix}$$



MATRIZ QUADRADA

$$\text{N}^{\circ} \text{ LINHAS} = \text{N}^{\circ} \text{ COLUNAS}$$

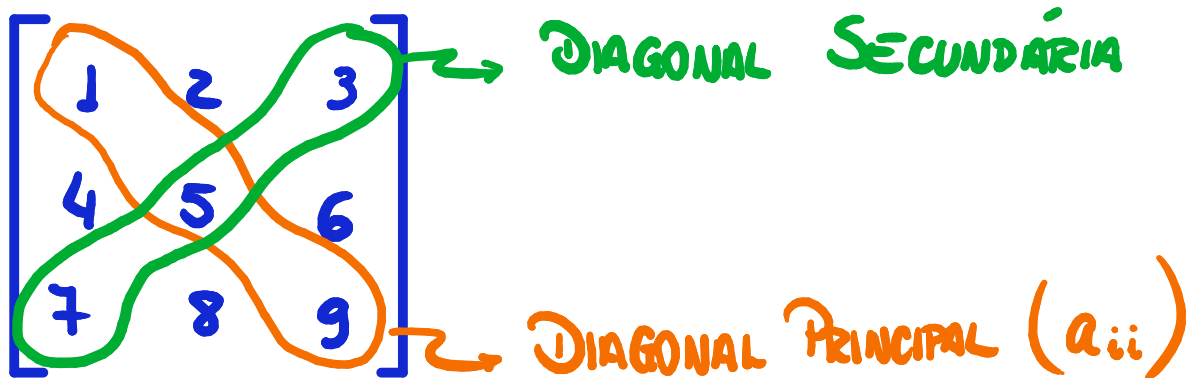
$$A = [1] \rightarrow \text{MATRIZ DE ORDEM 1}$$

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \rightarrow \text{MATRIZ DE ORDEM 2}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \rightarrow \text{MATRIZ DE ORDEM 3}$$



DIAGONAIS IMPORTANTES



TRAÇO DA MATRIZ

SOMA DOS ELEMENTOS DA DIAGONAL PRINCIPAL.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\text{tr}(A) = 1 + 5 + 9$$

$$\text{tr}(A) = 15$$



EXEMPLO

CALCULE O TRACO DA MATRIZ

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 4 & 2 & 6 \\ 1 & 8 & 1 \end{bmatrix}$$

$$\text{tr}(A) = 3 + 2 + 1$$

$$\text{tr } A = 6$$



EXEMPLO

CALCULE O TRAÇO DA MATRIZ DE ORDEM 4

DADA POR $a_{ij} = 2i \cdot j$

$$\text{tr } A = a_{11} + a_{22} + a_{33} + a_{44}$$

$$a_{11} = 2 \cdot 1 \cdot 1 = 2$$

$$a_{22} = 2 \cdot 2 \cdot 2 = 8$$

$$a_{33} = 2 \cdot 3 \cdot 3 = 18$$

$$a_{44} = 2 \cdot 4 \cdot 4 = 32$$

$$\text{tr } A = 2 + 8 + 18 + 32$$

$$\text{tr } A = 60$$



OPERAÇÕES ENTRE MATRIZES

IGUALDADE

$$A = B \leftrightarrow a_{ij} = b_{ij}$$

$$A = \begin{bmatrix} 3x & 4 \\ 2 & y \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & a \\ 2b & 4 \end{bmatrix}$$

$$A = B \rightarrow \begin{aligned} 3x &= 6 \\ x &= 2 \end{aligned}$$

$$a = 4$$

$$\begin{aligned} 2b &= 2 \\ b &= 1 \end{aligned}$$

$$y = 4$$

CONDIÇÃO: MATRIZES DE MESMA ORDEM



EXEMPLO

CALCULE OS VALORES DAS INCÓGNITAS :

$$\begin{bmatrix} 6 & k^2 \\ 2x+2 & y+4 \end{bmatrix} = \begin{bmatrix} 3a & 9 \\ 3x-4 & 2y \end{bmatrix}$$

$$3a = 6$$

$$a = 2$$

$$k^2 = 9$$

$$k = \pm 3$$

$$2x+2 = 3x-4$$

$$-x = -6$$

$$x = 6$$

$$y+4 = 2y$$

$$y = 4$$



SOMA E SUBTRAÇÃO

$$C = A \pm B \iff C_{ij} = a_{ij} \pm b_{ij}$$

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 2 \\ 7 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 5+6 & 2+2 \\ 3+7 & 6+1 \end{bmatrix} = \begin{bmatrix} 11 & 4 \\ 10 & 7 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 5-6 & 2-2 \\ 3-7 & 6-1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -4 & 5 \end{bmatrix}$$

CONDIÇÃO: MATRIZES DE MESMA ORDEM



PROPIEDADES

$$A + B = B + A$$

$$(A + B) + C = A + (B + C)$$

$$A + \underline{0} = A \quad \underline{0} : \text{MATRIZ NULA}$$

$$\text{tr}(A + B) = \text{tr} A + \text{tr} B$$

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}$$

$$\rightarrow \text{tr} A = 11$$

$$B = \begin{bmatrix} 6 & 2 \\ 7 & 1 \end{bmatrix}$$

$$\rightarrow \text{tr} B = 7$$

$$\left. \begin{array}{l} \text{tr} A = 11 \\ \text{tr} B = 7 \end{array} \right\} \text{tr} A + \text{tr} B = 18$$

$$A + B = \begin{bmatrix} 11 & 4 \\ 10 & 7 \end{bmatrix}$$

$$\rightarrow \text{tr}(A + B) = 18$$



EXEMPLO

CALCULE AS MATRIZES $A + B$ E $A - B$

$$A = \begin{bmatrix} 6 & 3 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 4 \\ 0 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 6 + 7 & 3 + 4 \\ 2 + 0 & 4 + 2 \end{bmatrix} = \begin{bmatrix} 13 & 7 \\ 2 & 6 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 6 - 7 & 3 - 4 \\ 2 - 0 & 4 - 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}$$



EXEMPLO

CALCULE O ELEMENTO DA LINHA 3 E COLUNA 4 DA MATRIZ C , SABENDO QUE

$$C = A - B, \quad a_{ij} = i + j \quad \text{e} \quad b_{ij} = 2i - j$$

$$C_{ij} = a_{ij} - b_{ij}$$

$$C_{34} = a_{34} - b_{34}$$

$$C_{34} = (3 + 4) - (2 \cdot 3 - 4)$$

$$C_{34} = 7 - 2$$

$$C_{34} = 5$$



MULTIPLICAÇÃO POR ESCALAR

$$B = \alpha \cdot A \leftrightarrow b_{ij} = \alpha \cdot a_{ij}$$

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}$$

$$3A = \begin{bmatrix} 3 \cdot 5 & 3 \cdot 2 \\ 3 \cdot 3 & 3 \cdot 6 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ 9 & 18 \end{bmatrix}$$

$$-5A = \begin{bmatrix} -5 \cdot 5 & -5 \cdot 2 \\ -5 \cdot 3 & -5 \cdot 6 \end{bmatrix} = \begin{bmatrix} -25 & -10 \\ -15 & -30 \end{bmatrix}$$



EXEMPLO

CALCULE AS MATRIZES $2A$ E $-7A$.

$$A = \begin{bmatrix} 6 & 3 \\ 2 & 4 \end{bmatrix}$$

$$2A = \begin{bmatrix} 2 \cdot 6 & 2 \cdot 3 \\ 2 \cdot 2 & 2 \cdot 4 \end{bmatrix} = \begin{bmatrix} 12 & 6 \\ 4 & 8 \end{bmatrix}$$

$$-7A = \begin{bmatrix} -7 \cdot 6 & -7 \cdot 3 \\ -7 \cdot 2 & -7 \cdot 4 \end{bmatrix} = \begin{bmatrix} -42 & -21 \\ -14 & -28 \end{bmatrix}$$



EXEMPLO

CALCULE O ELEMENTO m_{21} .

$$M + N = \begin{bmatrix} 3 & 7 \\ 6 & 12 \end{bmatrix}$$

$$M - N = \begin{bmatrix} 1 & 1 \\ -6 & 0 \end{bmatrix}$$

Solução # 1

$$M + N = \begin{bmatrix} 3 & 7 \\ 6 & 12 \end{bmatrix}$$

$$M - N = \begin{bmatrix} 1 & 1 \\ -6 & 0 \end{bmatrix}$$

$$2M = \begin{bmatrix} 4 & 8 \\ 0 & 12 \end{bmatrix}$$



$$M = \begin{bmatrix} 2 & 4 \\ 0 & 6 \end{bmatrix}$$

$m_{21} = 0$



Solução # 2

$$\begin{cases} m_{21} + n_{21} = 6 \\ m_{21} - n_{21} = -6 \end{cases}$$

(+)

$$2m_{21} = 0$$

$$m_{21} = 0$$



MULTIPLICAÇÃO ENTRE MATRIZES

$$A_{m \times n} \cdot B_{p \times q} = C_{m \times q}$$

Diagram illustrating matrix multiplication compatibility: A red bracket connects the n of $A_{m \times n}$ and the p of $B_{p \times q}$, with the text $n = p$ written below it. Green arrows show the flow from A and B to C , with an arrow pointing to the q in $C_{m \times q}$.

$$\begin{cases} A_{3 \times 2} \cdot B_{2 \times 4} = C_{3 \times 4} \\ B_{2 \times 4} \cdot A_{3 \times 2} = \nexists \end{cases}$$

Diagram illustrating non-commutativity: The first equation shows $A_{3 \times 2} \cdot B_{2 \times 4} = C_{3 \times 4}$ with a red bracket and $=$ under the dot. The second equation shows $B_{2 \times 4} \cdot A_{3 \times 2} = \nexists$ with a red bracket and $\neq$ under the dot.

$$\begin{cases} A_{3 \times 2} \cdot B_{2 \times 3} = C_{3 \times 3} \\ B_{2 \times 3} \cdot A_{3 \times 2} = D_{2 \times 2} \end{cases}$$

Diagram illustrating non-commutativity: The first equation shows $A_{3 \times 2} \cdot B_{2 \times 3} = C_{3 \times 3}$ with a red bracket and $=$ under the dot. The second equation shows $B_{2 \times 3} \cdot A_{3 \times 2} = D_{2 \times 2}$ with a red bracket and $=$ under the dot. A green arrow points from the $=$ in the second equation to the $=$ in the first equation.

$$\begin{cases} A_{3 \times 3} \cdot B_{3 \times 3} = C_{3 \times 3} \\ B_{3 \times 3} \cdot A_{3 \times 3} = D_{3 \times 3} \end{cases}$$

Diagram illustrating non-commutativity: The first equation shows $A_{3 \times 3} \cdot B_{3 \times 3} = C_{3 \times 3}$ with a red bracket and $=$ under the dot. The second equation shows $B_{3 \times 3} \cdot A_{3 \times 3} = D_{3 \times 3}$ with a red bracket and $=$ under the dot. Red text to the right states: $C \neq D$ (HÁ EXCEÇÕES).



EFETUANDO A MULTIPLICAÇÃO

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 1 \\ 3 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 2 & 3 \end{bmatrix} =$$

3×3 3×2

$$\begin{bmatrix} \frac{1 \cdot 1 + 2 \cdot 4 + 4 \cdot 2}{1 \cdot 2 + 2 \cdot 1 + 4 \cdot 3} \\ \frac{2 \cdot 1 + 1 \cdot 4 + 1 \cdot 2}{2 \cdot 1 + 1 \cdot 1 + 1 \cdot 3} \\ \frac{3 \cdot 1 + 1 \cdot 4 + 2 \cdot 2}{3 \cdot 2 + 1 \cdot 1 + 2 \cdot 3} \end{bmatrix} = \begin{bmatrix} 17 & 16 \\ 8 & 6 \\ 11 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} x & x \\ x & x \\ x & x \end{bmatrix}$$

3×2 3×2



PROPRIEDADES

$$A.B \neq B.A \quad (\text{HÁ EXCEÇÕES})$$

$$(A.B).C = A(B.C)$$

$$\alpha(A.B) = (\alpha.A).B = A(\alpha.B)$$

$$A(B+C) = A.B + A.C$$

$$(A+B)C = A.C + B.C$$

$$\left. \begin{array}{l} A.B + C.A \\ A(B+C) \end{array} \right\}$$

$$A.I = A \quad (I: \text{MATRIZ IDENTIDADE})$$

$$A.B = A.C \rightarrow B = C$$

$$A.B = \underline{0} \rightarrow A = \underline{0} \text{ ou } B = \underline{0}$$

$$\text{tr}(A.B) \checkmark = \text{tr}(B.A) \neq \text{tr} A . \text{tr} B$$



EXEMPLO

VERIFIQUE SE OS PRODUTOS ABAIXO SÃO POSSÍVEIS
E, EM CASO POSITIVO, EScreVA A ORDEM DA MATRIZ
RESULTANTE.

~~$$A_{2 \times 4} \cdot B_{2 \times 4} =$$~~

(A red arc under the dot is marked with a red $\neq$).

$$A_{3 \times 4} \cdot B_{4 \times 1} = C_{3 \times 1}$$

(A red arc under the dot is marked with a red $=$, and an orange arc is drawn below it).

$$A_{7 \times 1} \cdot B_{1 \times 6} = D_{7 \times 6}$$

(A red arc under the dot is marked with a red $=$, and an orange arc is drawn below it).

~~$$A_{5 \times 2} \cdot B_{3 \times 2} =$$~~

(A red arc under the dot is marked with a red $\neq$).



EXEMPLO

EFEETUE A MULTIPLICAÇÃO ABAIXO

$$\begin{matrix} 2 \times 3 \\ \begin{bmatrix} 4 & 2 & 3 \\ 0 & 5 & 1 \end{bmatrix} \end{matrix} \begin{matrix} 3 \times 2 \\ \begin{bmatrix} 1 & 3 \\ 0 & 4 \\ 5 & 7 \end{bmatrix} \end{matrix} = C_{2 \times 2}$$

$$\left[\begin{array}{c|c} 4 \cdot 1 + 2 \cdot 0 + 3 \cdot 5 & 4 \cdot 3 + 2 \cdot 4 + 3 \cdot 7 \\ \hline 0 \cdot 1 + 5 \cdot 0 + 1 \cdot 5 & 0 \cdot 3 + 5 \cdot 4 + 1 \cdot 7 \end{array} \right] = \begin{bmatrix} 19 & 41 \\ 5 & 27 \end{bmatrix}$$



EXEMPLO

SABENDO QUE $A^2 = A$ E $a \neq 0$, CALCULE b .

$$A = \begin{bmatrix} a & 0 \\ b & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ b & 1 \end{bmatrix} = \begin{bmatrix} a^2 + 0 \cdot b & a \cdot 0 + 0 \cdot 1 \\ ab + b \cdot 1 & b \cdot 0 + 1 \cdot 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a^2 & 0 \\ ab + b & 1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ b & 1 \end{bmatrix}$$

$$a^2 = a$$

$$a^2 - a = 0$$

$$a(a-1) = 0$$

~~$a = 0$~~ ou $a = 1$

$$ab + b = b$$

$$1 \cdot b = 0$$

$$b = 0$$



MATRIZES IMPORTANTES

MATRIZ LINHA

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 8 & 4 & 1 \end{bmatrix}$$

MATRIZ COLUNA

$$\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 7 \\ 8 \\ -1 \\ 0 \end{bmatrix}$$



MATRIZ NULA (0)

$$\underline{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} ; \quad \underline{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

LEMBRETE: $A + \underline{0} = \underline{A}$

MATRIZ TRIANGULAR

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

TRIANGULAR
SUPERIOR

$$\begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$$

TRIANGULAR
INFERIOR



MATRIZ DIAGONAL

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

MATRIZ IDENTIDADE (I_n)

$$\begin{bmatrix} 1 \end{bmatrix}$$

I_1

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

I_2

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

I_3

LEMBRETE : $A \cdot I = A$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



MATRIZ OPOTA (-A)

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$-A = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$$



MATRIZ IDEMPOTENTE

$$A^2 = A$$

CONSEQUÊNCIA:

$$A \cdot A^2 = A \cdot A$$

$$A^3 = A^2 \rightarrow A^3 = A$$

$$A \cdot A^3 = A \cdot A$$

$$A^4 = A^3 \rightarrow A^4 = A$$

$$A^k = A ; \forall k \in \mathbb{N}^*$$

$$\begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 9-6 & -18+12 \\ 3-2 & -6+4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}$$



MATRIZ NILPOTENTE

MATRIZ TAL QUE, PARA ALGUM $p \in \mathbb{N}^*$, TEM-SE:

$$A^p = \underline{0}$$

$$B = \begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \rightarrow B^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 4 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad C^2 = \begin{bmatrix} 0 & 0 & 12 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



MATRIZ TRANSPOSTA (A^T)

MATRIZ OBTIDA TROCANDO AS LINHAS PELAS COLUNAS.

$$A = \begin{bmatrix} 8 & 2 \\ 4 & 7 \\ 1 & 9 \end{bmatrix}_{3 \times 2} ; \quad A^T = \begin{bmatrix} 8 & 4 & 1 \\ 2 & 7 & 9 \end{bmatrix}_{2 \times 3}$$

PROPRIEDADES

$$(A^T)^T = A$$

$$(A + B)^T = A^T + B^T$$

$$(A \cdot B)^T = B^T \cdot A^T$$



MATRIZ SIMÉTRICA

$$\text{MATRIZ TAL QUE} \left\{ \begin{array}{l} A^T = A \\ a_{ij} = a_{ji} \end{array} \right.$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

OBS: DIAGONAL : $a_{11} = a_{11} \rightarrow a_{ii}$: QUALQUER

$$\begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix}$$



MATRIZ ANTI-SIMÉTRICA

$$\text{MATRIZ TAL QUE} \left\{ \begin{array}{l} A^T = -A \\ a_{ij} = -a_{ji} \end{array} \right.$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

OBS: DIAGONAL $\rightarrow a_{ii} = -a_{ii} \rightarrow \underline{a_{ii} = 0}$

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$



EXEMPLO

SE A É SIMÉTRICA, CALCULE $a + b + c$.

$$\begin{bmatrix} 1 & 2a & b+1 \\ 4 & 7 & 3c \\ 2b & 5c & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2a & b+1 \\ 4 & 7 & 3c \\ 2b & 5c & 4 \end{bmatrix}$$

$$2a = 4$$

$$a = 2$$

$$2b = b + 1$$

$$b = 1$$

$$3c = 5c$$

$$c = 0$$

$$a + b + c = 2 + 1 + 0 = 3$$



EXEMPLO

SE A É ANTI-SIMÉTRICA, CALCULE $m + n + p$

$$\begin{bmatrix} 0 & 2m & 0 \\ 4 & p+1 & 9-n \\ 0 & 2n & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2m & 0 \\ 4 & p+1 & 9-n \\ 0 & 2n & 0 \end{bmatrix}$$

$$p+1=0$$

$$p=-1$$

$$2m=-4$$

$$m=-2$$

$$2n=9-n$$

$$3n=9$$

$$n=3$$

$$m+n+p = -2+3+(-1) = 0$$



EXEMPLO

SE A E B SÃO SIMÉTRICAS, MOSTRE QUE

$$C = AB + BA^T \quad \text{E} \quad D = ABA^T$$

TAMBÉM SÃO SIMÉTRICAS.

$$A^T = A \quad ; \quad B^T = B$$

$$C = AB + BA^T$$

$$\begin{aligned} C^T &= (AB + BA^T)^T = (AB)^T + (BA^T)^T \\ &= B^T A^T + (A^T)^T B^T \\ &= B.A + A.B = AB + BA^T = C \\ C^T &= C \end{aligned}$$

$$D = A.B.A^T$$

$$\begin{aligned} D^T &= ((A.B)A^T)^T = (A^T)^T . (A.B)^T \\ &= A.B^T.A^T = A.B.A^T = D \\ D^T &= D \end{aligned}$$



MATRIZ INVERSA (A^{-1})

A MATRIZ A^{-1} , INVERSA DE A É TAL QUE

$$A \cdot A^{-1} = A^{-1} \cdot A = I$$

OBS # 1 : MATRIZES QUADRADAS

OBS # 2 : A INVERSA É ÚNICA

OBS # 3 : NEM TODA MATRIZ POSSUI INVERSA

PROPRIEDADES

$$(A + B)^{-1} = A^{-1} + B^{-1}$$

$$(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$$



AS MATRIZES ABAIXO SÃO INVERSAS ?

$$\begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix} = \begin{bmatrix} 7 \cdot 3 + 4(-5) & 7(-4) + 4 \cdot 7 \\ 5 \cdot 3 + 3(-5) & 5(-4) + 3 \cdot 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

SIM, ELAS SÃO INVERSAS



COMO OBTENER A INVERSA ?

$$A = \begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix} ; A^{-1} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$A \cdot A^{-1} = I$$

$$\begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 3a+5b & 3c+5d \\ 2a+3b & 2c+3d \end{bmatrix}$$

$$\begin{bmatrix} 3a+5b & 3c+5d \\ 2a+3b & 2c+3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} 3a + 5b = 1 \\ 2a + 3b = 0 \end{cases}$$

$$\begin{cases} 3c + 5d = 0 \\ 2c + 3d = 1 \end{cases}$$



$$\begin{cases} 3a + 5b = 1 & (2) \\ 2a + 3b = 0 & (-3) \end{cases} \rightarrow \begin{cases} 6a + 10b = 2 \\ -6a - 9b = 0 \end{cases}$$

$$\ominus$$

$$\begin{aligned} b &= 2 \\ a &= -3 \end{aligned}$$

$$\begin{cases} 3c + 5d = 0 & (2) \\ 2c + 3d = 1 & (-3) \end{cases} \rightarrow \begin{cases} 6c + 10d = 0 \\ -6c - 9d = -3 \end{cases}$$

$$\ominus$$

$$2c - 9 = 1$$

$$c = 5$$

$$d = -3$$

$$c = 5$$

$$A^{-1} = \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 2 & -3 \end{bmatrix}$$



EXEMPLO

ENCONTRE A INVERSA DE $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 5a + 2b & 5c + 2d \\ 3a + b & 3a + d \end{bmatrix}$$

$$\begin{bmatrix} 5a + 2b & 5c + 2d \\ 3a + b & 3c + d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} 5a + 2b = 1 \\ 3a + b = 0 \end{cases}$$

$$\begin{cases} 5c + 2d = 0 \\ 3c + d = 1 \end{cases}$$



$$\begin{cases} 5a + 2b = 1 \\ 3a + b = 0 \quad (-2) \end{cases}$$

$$\rightarrow 3(-1) + b = 0$$

$$b = 3$$

$$\begin{cases} 5a + 2b = 1 \\ -6a - 2b = 0 \end{cases}$$

⊕

$$-a = 1 \rightarrow \underline{a = -1}$$

$$\underline{b = 3}$$

$$\begin{cases} 5c + 2d = 0 \\ 3c + d = 1 \quad (-2) \end{cases}$$

$$\rightarrow 3 \cdot 2 + d = 1$$

$$d = -5$$

$$\begin{cases} 5c + 2d = 0 \\ -6c - 2d = -2 \end{cases}$$

⊕

$$-c = -2$$

$$\underline{c = 2}$$

$$\underline{d = -5}$$

$$A^{-1} = \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$$



EXEMPLO

ENCONTRE A SOMA DOS ELEMENTOS DA PRIMEIRA COLUNA DA INVERSA DA MATRIZ:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 5 & 8 & 4 & 1 \\ 7 & 9 & 5 & 1 \\ 5 & 5 & 6 & 2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 5 & 8 & 4 & 1 \\ 7 & 9 & 5 & 1 \\ 5 & 5 & 6 & 2 \end{bmatrix}}_A \underbrace{\begin{bmatrix} a & x & x & x \\ b & x & x & x \\ c & x & x & x \\ d & x & x & x \end{bmatrix}}_{A^{-1}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_I$$

$$\underline{a + b + c + d = 1}$$



EXEMPLO

Se A é INVERSÍVEL, CALCULE X .

$$A \cdot (B^{-1} \cdot X) = C^{-1} \cdot A$$

$$A \cdot (B^{-1} \cdot X) = C^{-1} \cdot A$$

$$A^{-1} \cdot A \cdot B^{-1} \cdot X = A^{-1} \cdot C^{-1} \cdot A$$

$$I \cdot B^{-1} \cdot X = A^{-1} \cdot C^{-1} \cdot A$$

$$B \cdot B^{-1} \cdot X = B \cdot A^{-1} \cdot C^{-1} \cdot A$$

$$I \cdot X = B \cdot A^{-1} \cdot C^{-1} \cdot A$$

$$X = B \cdot A^{-1} \cdot C^{-1} \cdot A$$



MATRIZ ORTOGONAL

MATRIZ TAL QUE $A^{-1} = A^T$

$$A = \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix} ; A^T = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}}_A \underbrace{\begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}}_{A^T} =$$

$$\begin{bmatrix} (1/2)^2 + (\sqrt{3}/2)^2 & 0 \\ 0 & (-\sqrt{3}/2)^2 + (1/2)^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A \cdot A^T = I \rightarrow$$



$$A^T = A^{-1}$$

EXEMPLO

MOSTRE QUE A MATRIZ ABAIXO É ORTOGONAL

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$R^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$R \cdot R^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta(-\sin\theta) + \cos\theta\sin\theta \\ \cos\theta(-\sin\theta) + \cos\theta\sin\theta & (-\sin\theta)^2 + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$R^T = R^{-1}$$