Binômio de Newton

$$(a+b)^0 = 1$$

$$(a+b)^1 = 1a + 1b$$

$$(a+b)^2 = 1a^2 + 2ab + 1b^2$$

$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$$

$$(a+b)^4 = \underbrace{1a^4}_{T_0} + \underbrace{4a^3b}_{T_1} + \underbrace{6a^2b^2}_{T_2} + \underbrace{4ab^3}_{T_3} + \underbrace{1b^4}_{T_4}$$

- Triângulo de Pascal

Linha 0: 1
Linha 1: 1 1
Linha 2: 1 2 1
Linha 3: 1 3 3 1
Linha 4: 1 4 6 4 1
Linha 5: 1 5 10 10 5 1

$$\binom{0}{0}$$
$$\binom{1}{0} \quad \binom{1}{1}$$
$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$
$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\boxed{(a+b)^n = \sum_{k=0}^{n} \underbrace{\binom{n}{k} a^{n-k} b^k}_{T_k}}$$

Obs.: Relação de Stifel

$$\binom{n}{p} + \binom{n}{p+1} = \binom{n+1}{p+1}$$

$$\binom{4}{0} + \binom{4}{1} = \binom{5}{1}$$

Exemplo 1: Qual o termo independente de x na expansão de

$$\left(x^2 + \frac{3}{x}\right)^{12}$$

$$T_k = \binom{n}{k} a^{n-k} b^k = \binom{12}{k} (x^2)^{12-k} \cdot \left(\frac{3}{x}\right)^k$$

$$= \binom{12}{k} x^{24-2k} \cdot \frac{3^k}{x^k} = \binom{12}{k} x^{24-3k} \cdot 3^k$$

Se $k=8$, temos $T_8 = \binom{12}{8} \cdot 3^8 = 495 \cdot 3^8$

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Exemplo 2: (ITA)

Determine o valor da soma

$$\binom{4n}{0}-\binom{4n}{2}+\binom{4n}{4}-\dots-\binom{4n}{4n-2}+\binom{4n}{4n}$$

$$(1+i)^{4n}=\sum_{k=0}^{4n}\binom{4n}{k}\cdot i^k$$

$$=\binom{4n}{0}+\binom{4n}{1}i-\binom{4n}{2}-\binom{4n}{3}i+\dots+\binom{4n}{4n}$$

$$Re\{(1+i)^{4n}\}=Re\{((1+i)^4)^n\}=Re\{(-4)^n\}=(-4)^n$$

$$(1+i)^4=1^4+4\cdot 1^3\cdot i+6\cdot 1^2\cdot i^2+4\cdot 1\cdot i^3+i^4$$
$$=1+\cancel{4i}-6-\cancel{4i}+1=-4$$

- Expansão Multinomial (Polinômio de Leibniz)

$$(a+b+c+\dots+z)^n$$

$$= \underbrace{\sum_{\alpha+\beta+\dots+\varsigma=n} \frac{n!}{\alpha!\beta!\gamma!\dots\varsigma!}\, a^{\alpha}\cdot b^{\beta}\cdot c^{\gamma}\dots z^{\varsigma}}_{T}$$

Qual o termo independente da expansão de

$$\left(1+x+\frac{1}{x^2}\right)^3 \qquad \alpha+\beta+\gamma=3 \qquad \boxed{\begin{matrix}\alpha=0\\ \beta=2\\ \gamma=1\end{matrix}}$$

$$T=\frac{3!}{\alpha!\beta!\gamma!}\cdot 1^{\alpha}\cdot x^{\beta}\cdot\left(\frac{1}{x^2}\right)^{\gamma}=\frac{3!}{\alpha!\beta!\gamma!}\cdot x^{\beta-2\gamma}$$

$$=\frac{3!}{0!2!1!}\cdot x^0 = 3$$

Exemplo 1: (ITA-2013)

Qual o coeficiente de x^4y^4 no desenvolvimento de

$$(1+x+y)^{10} \qquad \alpha = 2 \quad \beta = 4 \quad \gamma = 4$$

$$T = \frac{10!}{\alpha!\,\beta!\,\gamma!} \cdot 1^{\alpha} \cdot x^{\beta} y^{\gamma}$$

$$= \frac{10!}{2!\,4!\,4!} \cdot x^4 y^4 = \frac{10 \cdot 9 \cdot \cancel{8} \cdot 7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4!}}{\cancel{2} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{4!}}\, x^4 y^4$$

$$= \boxed{3150}\, x^4 y^4$$