

DETERMINANTES

CONCEITO

DETERMINANTE É UM ESCALAR ASSOCIADO A MATRIZES QUADRADAS.

NOTAÇÃO

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\det A = |A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$



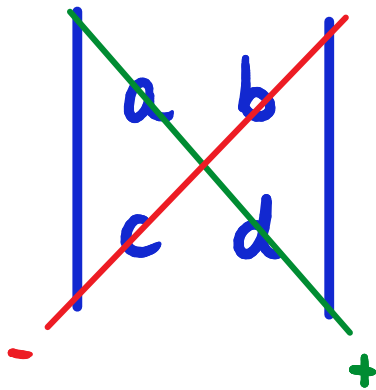
MATRIZ DE ORDEN 1

$$A = [k] \rightarrow \boxed{\det A = k}$$

$$A = [7] \rightarrow \det A = 7$$

MATRIZ DE ORDEN 2

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a.d - b.c$$



$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1.4 - 2.3 = -2$$



MATRIZ DE ORDEM 3

* REGRA DE SARRUS *

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

det A =

1	2	3	1	2
4	5	6	4	5
7	8	9	7	8

- - - + + +

$$\det A = (1 \cdot 5 \cdot 9 + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8) - (3 \cdot 5 \cdot 7 + 1 \cdot 6 \cdot 8 + 2 \cdot 4 \cdot 9)$$

$$\det A = (45 + 84 + 96) - (105 + 48 + 72)$$

$$\det A = 225 - 225$$

$\det A = 0$

EXEMPLO

CALCULE O DETERMINANTE :

$$\begin{vmatrix} 6 & 7 \\ 9 & 5 \end{vmatrix}$$

$$\begin{vmatrix} 6 & 7 \\ 9 & 5 \end{vmatrix} = 6 \cdot 5 - 7 \cdot 9 = 30 - 63 = -33$$



EXEMPLO

CALCULE O DETERMINANTE :

$$\begin{vmatrix} 2 & 1 & 3 \\ -4 & 5 & -2 \\ 7 & 3 & 1 \end{vmatrix}$$

A 3x3 matrix is shown with orange numbers: $\begin{vmatrix} 2 & 1 & 3 \\ -4 & 5 & -2 \\ 7 & 3 & 1 \end{vmatrix}$. Red diagonal lines cross the matrix from top-left to bottom-right and from top-right to bottom-left. A dashed vertical line is drawn between the first and second columns. The matrix is followed by an equals sign.

$$= (2 \cdot 5 \cdot 1 + 1(-2)7 + 3(-4)3) - (3 \cdot 5 \cdot 7 + 2(-2) \cdot 3 + 1(-4) \cdot 1)$$

$$= (10 - 14 - 36) - (105 - 12 - 4)$$

$$= -40 - (89) = -129$$



EXEMPLO

RESOLVA A EQUAÇÃO

$$\begin{vmatrix} x & 5 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & x \end{vmatrix} = 24$$

$$\begin{vmatrix} x & 5 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & x \end{vmatrix} = (2x^2 + 30 + 36) - (6 + 18x + 20x)$$

$$= 2x^2 + 66 - (38x + 6) = \underline{2x^2 - 38x + 60}$$

$$2x^2 - 38x + 60 = 24$$

$$2x^2 - 38x + 36 = 0$$

$$x^2 - 19x + 18 = 0$$

$$S = 19$$

$$P = 18$$

$$x = 1 \text{ ou } x = 18$$

$$x = 18$$



MATRIZES DE ORDEM $n \geq 2$

MENOR COMPLEMENTAR (d_{ij})

O MENOR COMPLEMENTAR d_{ij} DE UMA MATRIZ É O DETERMINANTE DA MATRIZ OBTIDA AO RETIRAR A LINHA i E A COLUNA j .



$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$\rightarrow$

$$d_{11} = \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix}$$

$$d_{11} = 5 \cdot 9 - 8 \cdot 6$$

$$d_{11} = -3$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$\rightarrow$

$$d_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix}$$

$$d_{23} = 8 - 14$$

$$d_{23} = -6$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$\rightarrow$

$$d_{31} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}$$

$$d_{31} = 12 - 15$$

$$d_{13} = -3$$



EXEMPLO

CALCULE OS MENORES COMPLEMENTARES d_{13} E d_{32} DA MATRIZ ABAIXO.

$$\begin{bmatrix} 1 & 2 & -2 \\ 4 & -1 & 3 \\ 3 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -2 \\ 4 & -1 & 3 \\ 3 & 1 & -1 \end{bmatrix}$$

$$d_{13} = \begin{vmatrix} 4 & -1 \\ 3 & 1 \end{vmatrix} = 4 - (-3)$$

$$\underline{d_{13} = 7}$$

$$\begin{bmatrix} 1 & 2 & -2 \\ 4 & -1 & 3 \\ 3 & 1 & -1 \end{bmatrix}$$

$$d_{32} = \begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix} = 3 - (-8)$$

$$\underline{d_{32} = 11}$$



COFATOR OU COMPLEMENTO ALGÉBRICO (a_{ij}^*)

$$a_{ij}^* = (-1)^{i+j} \cdot d_{ij}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$a_{11}^* = (-1)^{1+1} \cdot d_{11}$$

$$\rightarrow d_{11} = \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} = 45 - 48 = -3$$

$$a_{11}^* = (-1)^2 \cdot (-3)$$

$$a_{11}^* = -3$$



$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$a_{23}^* = (-1)^{2+3} \cdot d_{23}$$

$$d_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = 8 - 14$$

$$d_{23} = -6$$

$$a_{23}^* = (-1)^{2+3} \cdot (-6)$$

$$a_{23}^* = 6$$



MATRIZ COFATORA (A^*)

É A MATRIZ CUJOS ELEMENTOS SÃO OS COFADORES DE UMA MATRIZ.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A^* = \begin{bmatrix} a_{11}^* & a_{12}^* & a_{13}^* \\ a_{21}^* & a_{22}^* & a_{23}^* \\ a_{31}^* & a_{32}^* & a_{33}^* \end{bmatrix}$$



EXEMPLO

CALCULE OS COFATORES a_{22}^* E a_{31}^* .

$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$a_{ij}^* = (-1)^{i+j} \cdot d_{ij}$$



$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$a_{22}^* = (-1)^{2+2} \cdot d_{22}$$

$$d_{22} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 - 2$$

$$d_{22} = -1$$

$$a_{22}^* = (-1)^{2+2} \cdot (-1)$$

$$a_{22}^* = -1$$

$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$a_{31}^* = (-1)^{3+1} \cdot d_{31}$$

$$d_{31} = \begin{vmatrix} 3 & 1 \\ -1 & 0 \end{vmatrix} = 0 - (-1) = 1$$

$$a_{31}^* = (-1)^{3+1} \cdot 1$$

$$a_{31}^* = 1$$



TEOREMA DE LAPLACE

O DETERMINANTE DE UMA MATRIZ DE ORDEM $n \geq 2$ É DADO PELA SOMA DOS PRODUTOS DOS ELEMENTOS DE UMA FILA PELOS SEUS RESPECTIVOS COFATORES.

$$\begin{vmatrix} 1 & 2 & 0 & 3 \\ 3 & 2 & 0 & 1 \\ 1 & 0 & 1 & 4 \\ 2 & 4 & 0 & 1 \end{vmatrix}$$



$$\begin{vmatrix} 1 & 2 & 0 & 3 \\ 3 & 2 & 0 & 1 \\ 1 & 0 & 1 & 4 \\ 2 & 4 & 0 & 1 \end{vmatrix} =$$

$$\det A = a_{21} \cdot a_{21}^* + a_{22} \cdot a_{22}^* + a_{23} \cdot a_{23}^* + a_{24} \cdot a_{24}^*$$

$$\det A =$$

$$3 \cdot (-1)^{2+1} \cdot \begin{vmatrix} 2 & 0 & 3 \\ 0 & 1 & 4 \\ 4 & 0 & 1 \end{vmatrix} + 2 \cdot (-1)^{2+2} \cdot \begin{vmatrix} 1 & 0 & 3 \\ 1 & 1 & 4 \\ 2 & 0 & 1 \end{vmatrix} +$$

$$+ 0 \cdot (-1)^{2+3} \cdot \begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 4 \\ 2 & 4 & 1 \end{vmatrix} + 1 \cdot (-1)^{2+4} \cdot \begin{vmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 2 & 4 & 0 \end{vmatrix}$$



$$\det A = 3 \cdot (-1)^{2+1} \cdot (-10) + 2 \cdot (-1)^{2+2} \cdot (-5) + \\ + 0 \cdot (-1)^{2+3} \cdot (10) + 1 \cdot (-1)^{2+4} \cdot (0)$$

$$\det A = 30 + (-10) + 0 + 0$$

$$\det A = 20$$



ESCOLHENDO OUTRA FILA ...

$$\begin{vmatrix} 1 & 2 & 0 & 3 \\ 3 & 2 & 0 & 1 \\ 1 & 0 & 1 & 4 \\ 2 & 4 & 0 & 1 \end{vmatrix} =$$

$$\det A = \cancel{a_{13}} \cdot a_{13}^* + \cancel{a_{23}} \cdot a_{23}^* + a_{33} \cdot a_{33}^* + \cancel{a_{43}} \cdot a_{43}^*$$

$$\det A = a_{33} \cdot a_{33}^*$$

$$\det A = 1 \cdot (-1)^{3+3} \cdot \begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 4 & 1 \end{vmatrix}$$

$$\det A = 20$$



Os Segredos ...

1 - ESCOLHER A MELHOR FILA
(MAIS ZEROS)

2 - MANIPULAR A MATRIZ

↳ * T. JACOBI



EXEMPLO

CALCULE O DETERMINANTE ABAIXO :

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 0 & 0 \\ 2 & 1 & 1 & 3 \\ 4 & 0 & 1 & 2 \end{vmatrix}$$

$$\det A = \cancel{a_{21} \cdot a_{21}^*} + a_{22} \cdot a_{22}^* + \cancel{a_{23} \cdot a_{23}^*} + \cancel{a_{24} \cdot a_{24}^*}$$

$$\det A = a_{22} \cdot a_{22}^*$$



$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 0 & 0 \\ 2 & 1 & 1 & 3 \\ 4 & 0 & 1 & 2 \end{vmatrix} = a_{22} \cdot a_{22}^*$$

$$= 2 \cdot (-1)^{2+2} \cdot \begin{vmatrix} 1 & 3 & 4 & 1 & 3 \\ 2 & 1 & 3 & 2 & 1 \\ 4 & 1 & 2 & 4 & 1 \end{vmatrix}$$

$$= 2 \cdot 1 \cdot \left((2 + 36 + 8) - (16 + 3 + 12) \right)$$

$$= 2 \cdot (46 - 31)$$

$$= 2 \cdot 15$$

$$= 30$$



TEOREMA

SEJAM MATRIZES A, B E C QUE POSSUAM APENAS 1 FILA DISTINTA, DA FORMA:

$$\begin{vmatrix} a & b & c \\ x & y & z \\ d & e & f \end{vmatrix} = \det A$$

$$\begin{vmatrix} a & b & c \\ u & v & w \\ d & e & f \end{vmatrix} = \det B$$

$$\begin{vmatrix} a & b & c \\ x+u & y+v & z+w \\ d & e & f \end{vmatrix} = \det C$$

ENTÃO:

$$\det C = \det A + \det B$$

OBS: VÁLIDO PARA MATRIZES DE QUALQUER ORDEM



DEM.:

COFATORES DA FILA DISTINTA:

$$\det A = x \cdot a_{z1}^* + y \cdot a_{z2}^* + z \cdot a_{z3}^*$$



$$\det B = u \cdot a_{z1}^* + v \cdot a_{z2}^* + w \cdot a_{z3}^*$$

$$\det C = (x+u) a_{z1}^* + (y+v) a_{z2}^* + (z+w) a_{z3}^*$$

$$= \underbrace{x \cdot a_{z1}^* + y \cdot a_{z2}^* + z \cdot a_{z3}^*}_{\det A} +$$

$$+ \underbrace{u \cdot a_{z1}^* + v \cdot a_{z2}^* + w \cdot a_{z3}^*}_{\det B}$$

$$\det C = \det A + \det B$$



$$\begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = 5$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} = 8$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ x+p & y+q & z+r \end{vmatrix} = 5 + 8 = 13$$



EXEMPLO

CALCULE SEM DESENVOLVER :

$$\begin{vmatrix} 2 & 1 & 3 \\ 1 & 3 & 4 \\ 4 & 1 & 2 \end{vmatrix} = -15$$

$$\begin{vmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 4 & 1 & 3 \end{vmatrix} = -5$$

$$\begin{vmatrix} 2 & 1 & 5 \\ 1 & 3 & 5 \\ 4 & 1 & 5 \end{vmatrix} = ?$$

$$\begin{vmatrix} 2 & 1 & 5 \\ 1 & 3 & 5 \\ 4 & 1 & 5 \end{vmatrix} = -15 + (-5) = -20$$



$$\begin{vmatrix} 2 & 1 & 5 \\ 1 & 3 & 5 \\ 4 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 3+2 \\ 1 & 3 & 4+1 \\ 4 & 1 & 2+3 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 1 & 3 \\ 1 & 3 & 4 \\ 4 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 4 & 1 & 3 \end{vmatrix}$$

$$= -15 + (-5)$$

$$= -20$$



COMBINAÇÃO LINEAR

GENERICAMENTE

* FUNÇÕES : $f(x) = 4.g(x) + 7.h(x)$

* LINHAS : $L_2 = -2L_1 + 4.L_3 + 6.L_7$

* COLUNAS : $C_6 = 2C_1 + 3C_3 - C_4$

$$\left[\begin{array}{ccc} 1 & 3 & 1 \\ 6 & 7 & 4 \\ 4 & 1 & 2 \end{array} \right] \quad \left. \begin{array}{l} 6 = 1.4 + 2.1 \\ 7 = 1.1 + 2.3 \\ 4 = 1.2 + 2.1 \end{array} \right\}$$

$$L_2 = 1.L_3 + 2L_1$$



TEOREMA DE JACOBI

SUBSTITUIR UMA FILA PELA
SOMA DESSA FILA
COM UMA
COMBINAÇÃO LINEAR DE FILAS PARALELAS
NÃO ALTERA O DETERMINANTE.



$$\det A = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 2 & 1 \\ 2 & 1 & 1 & 3 \\ 4 & 0 & 1 & 2 \end{vmatrix} \quad L_2 = L_2 - 3L_1$$

$$\det A = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -7 & -11 \\ 2 & 1 & 1 & 3 \\ 4 & 0 & 1 & 2 \end{vmatrix} \quad L_3 = L_3 - 2.L_1$$

$$\det A = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -7 & -11 \\ 0 & -3 & -5 & -5 \\ 4 & 0 & 1 & 2 \end{vmatrix} \quad L_4 = L_4 - 4.L_1$$

$$\det A = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -7 & -11 \\ 0 & -3 & -5 & -5 \\ 0 & -8 & -11 & -14 \end{vmatrix}$$

$$\det A = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 2 & 1 \\ 2 & 1 & 1 & 3 \\ 4 & 0 & 1 & 2 \end{vmatrix} \quad \begin{array}{l} L_2 = L_2 - 3L_1 \\ L_3 = L_3 - 2L_1 \\ L_4 = L_4 - 4L_1 \end{array}$$

$$\det A = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -7 & -11 \\ 0 & -3 & -5 & -5 \\ 0 & -8 & -11 & -14 \end{vmatrix}$$



EXEMPLO

PREPARE A MATRIZ PARA QUE SEJA
FÁCIL APLICAR O TEOREMA DE LAPLACE.

$$\begin{vmatrix} 2 & 1 & 5 & 4 \\ 1 & 2 & 2 & 1 \\ 4 & 5 & 3 & 2 \\ 1 & 1 & 2 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 1 & 5 & 4 \\ 1 & 2 & 2 & \textcircled{1} \\ 4 & 5 & 3 & 2 \\ 1 & 1 & 2 & 2 \end{vmatrix} \begin{array}{l} L_1 = L_1 - 4L_2 \\ L_3 = L_3 - 2L_2 \\ L_4 = L_4 - 2L_2 \end{array}$$

$$\begin{vmatrix} -2 & -7 & -3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & -1 & 0 \\ -1 & -3 & -2 & 0 \end{vmatrix}$$



PROPIEDADES

* P.1 - FILAS COM ZEROS:

$$\det = A$$

$$\begin{vmatrix} 0 & 0 & 0 \\ 6 & 7 & 4 \\ 4 & 1 & 2 \end{vmatrix} = 0 \cdot a_{11}^* + 0 \cdot a_{12}^* + 0 \cdot a_{13}^* = 0$$



* P.Z - FILAS PARALELAS IGUAIS:

$$\det = 0$$

$$\begin{vmatrix} 4 & 1 & 2 \\ 6 & 7 & 4 \\ 4 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ 6 & 7 & 4 \\ 4 & 1 & 2 \end{vmatrix} = 0$$

$L_1 = L_1 - L_3$



* P.3 - FILAS PARALELAS PROPORCIONAIS

$$\det = 0$$

$$\begin{vmatrix} 1 & 3 & 2 \\ 3 & 9 & 6 \\ 4 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 2 \\ 0 & 0 & 0 \\ 4 & 1 & 2 \end{vmatrix}$$

$L_2 = L_2 - 3L_1$



* P.4 - FILA COMBINAÇÃO LINEAR DE
OUTRAS FILAS PARALELAS :

$$\det = 0$$

$$\begin{vmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 5 & 12 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{vmatrix} =$$

$$L_3 = L_3 - 2L_1 - 3L_2$$



* P. 5 - TROCA DE FILAS PARALELAS:

DETERMINANTE INVERTE O SINAL

$$\begin{vmatrix} 1 & 3 & 2 \\ 5 & 2 & 1 \\ 4 & 1 & 2 \end{vmatrix} = -21$$

$$\begin{vmatrix} 5 & 2 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 2 \end{vmatrix} = 21$$



* P.6 - MULTIPLICAR FILA POR K:

DETERMINANTE MULTIPLICA POR K.

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \cdot a_{11}^* + b \cdot a_{12}^* + c \cdot a_{13}^* = \det A$$

$$\begin{vmatrix} K \cdot a & K \cdot b & K \cdot c \\ d & e & f \\ g & h & i \end{vmatrix} = K \cdot a \cdot a_{11}^* + K \cdot b \cdot a_{12}^* + K \cdot c \cdot a_{13}^* \\ = K (a \cdot a_{11}^* + b \cdot a_{12}^* + c \cdot a_{13}^*) \\ = \underline{K \cdot \det A}$$

$$\begin{vmatrix} 2 & 1 & 2 \\ 4 & 1 & 3 \\ 1 & 3 & 1 \end{vmatrix} = 5$$

$\times 3$

$$\begin{vmatrix} 6 & 3 & 6 \\ 4 & 1 & 3 \\ 1 & 3 & 1 \end{vmatrix} = 3 \cdot 5 = 15;$$

$\times 2$

$$\begin{vmatrix} 6 & 6 & 6 \\ 4 & 2 & 3 \\ 1 & 6 & 1 \end{vmatrix} = 2 \cdot 15 = 30$$



* P.7 - MULTIPLICAR MATRIZ DE ORDEN n POR K:

$$\det(K.A) = K^n \cdot \det A$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \det A ;$$

A

$$\begin{vmatrix} Ka & Kb \\ Kc & Kd \end{vmatrix} = K^2 \cdot \det A$$

KA

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \det A ;$$

A

$$\begin{vmatrix} Ka & Kb & Kc \\ Kd & Ke & Kf \\ Kg & Kh & Ki \end{vmatrix} = K^3 \cdot \det A$$

KA

$$\begin{vmatrix} 2 & 1 & 2 \\ 4 & 1 & 3 \\ 1 & 3 & 1 \end{vmatrix} = 5$$

$$\begin{vmatrix} 20 & 10 & 20 \\ 40 & 10 & 30 \\ 10 & 30 & 10 \end{vmatrix} = 10^3 \cdot 5 = 5.000$$



* P.8 - MATRIZ TRANSPOSTA

$$\det(A^T) = \det(A)$$

* P.9 - MATRIZ TRIANGULAR

$$\begin{vmatrix} x & a & b & c \\ 0 & y & d & e \\ 0 & 0 & w & f \\ 0 & 0 & 0 & z \end{vmatrix} = x \cdot y \cdot w \cdot z$$

* P.10 - TEOREMA DE BINET

$$\det(A \cdot B) = \det A \cdot \det B$$



* P.11 - MATRIZ IDENTIDADE

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1^4 = 1 ;$$

$$\det I_n = 1$$

* P.12 - MATRIZ INVERSA

$$\det A^{-1} = \frac{1}{\det A}$$

A SÓ TEM
INVERSA

SE $\det A \neq 0$

$$A \cdot A^{-1} = I$$

$$\det(A \cdot A^{-1}) = \det I$$

$$\det A \cdot \det A^{-1} = 1$$

$$\det A^{-1} = \frac{1}{\det A}$$

EXEMPLO

$$\begin{vmatrix} 5 & 7 & 2 \\ 0 & 0 & 0 \\ 4 & 1 & 2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 5 & 7 & 2 \\ 5 & 7 & 2 \\ 4 & 1 & 3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 8 & 2 & 6 \\ 5 & 7 & 2 \\ 4 & 1 & 3 \end{vmatrix} = 0 \quad \begin{array}{l} \text{(LINHAS PROPORCIONAIS)} \\ L_1 = 2 \cdot L_3 \end{array}$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ 2a+3d & 2b+3e & 2c+3f \end{vmatrix} = 0$$

$$L_3 = 2L_1 + 3L_2$$



EXEMPLO

$$\begin{vmatrix} 2 & b & 6 & 1 \\ a & 3 & 1 & 2 \\ 3 & 5 & 0 & d \\ 7 & 2 & c & 4 \end{vmatrix} = 5 ; \quad \begin{vmatrix} 4 & -b & 18 & 4 \\ 2a & -3 & 3 & 8 \\ 6 & -5 & 0 & 4d \\ 14 & -2 & 3c & 16 \end{vmatrix} = ?$$

$$\begin{vmatrix} 4 & -b & 18 & 4 \\ 2a & -3 & 3 & 8 \\ 6 & -5 & 0 & 4d \\ 14 & -2 & 3c & 16 \end{vmatrix} = 5 \cdot 2 \cdot (-1) \cdot 3 \cdot 4 = -120$$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $C_1 \quad C_2 \quad C_3 \quad C_4$



EXEMPLO

SEJAM A, B E C MATRIZES INVERTÍVEIS DE ORDEM 4. CALCULE $\det A$ SABENDO QUE:

$$\bullet A \cdot B = C^{-1} \quad \bullet A = 3B \quad \bullet \det C = \frac{1}{3}$$

$$A \cdot B = C^{-1}$$

$$\det(A \cdot B) = \det(C^{-1})$$

$$\det A \cdot \det B = 3$$

$$3^4 \cdot \det B \cdot \det B = 3$$

$$(\det B)^2 = \frac{1}{27}$$

$$\det B = \pm \sqrt{\frac{1}{27}}$$

$$\det B = \pm \frac{\sqrt{3}}{9}$$

$$\downarrow$$
$$\det C^{-1} = 3$$

$$\det A = 3^4 \cdot \det B$$

$$\det A = 3^4 \cdot \pm \frac{\sqrt{3}}{9}$$

$$\det A = \pm 9\sqrt{3}$$



EXEMPLO

SEJA A UMA MATRIZ INVERSÍVEL DE ORDEN 6.

SE $A^4 + 2A = \underline{0}$, CALCULE $\det A$.

$$A^4 + 2A = \underline{0}$$

$$A^4 = \underline{0} - 2A$$

$$A^4 = -2A$$

$$\det(A^4) = \det(-2A)$$

$$(\det A)^4 = (-2)^6 \cdot \det A$$

$$x^4 = 2^6 \cdot x$$

$$x^4 - 2^6 x = 0$$

$$x(x^3 - 2^6) = 0$$

~~$$x = 0$$~~

$$\text{ou}$$
$$\sqrt[3]{x^3} = \sqrt[3]{2^6}$$

$$x = 2^2$$

$$\boxed{\det A = 4}$$



EXEMPLO

SE A É ANTI-SIMÉTRICA DE ORDEM ÍMPAR,
CALCULE SEU DETERMINANTE.

$$A^T = -A$$

$$\det(A^T) = \det(-A)$$

$$\det A = (-1)^n \cdot \det A$$

$$(-1)^n = \begin{cases} -1; & n \text{ ÍMPAR} \\ 1; & n \text{ PAR} \end{cases}$$

$$\det A = -\det A$$

$$2\det A = 0$$

$$\boxed{\det A = 0}$$



EXEMPLO

CALCULE $\det A = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & a+c & a+b \end{vmatrix}$

$$L_3 = L_3 + L_2$$

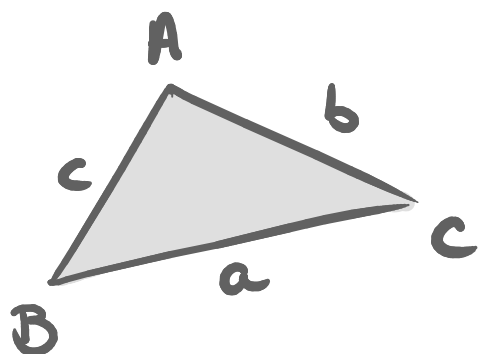
$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & a+c & a+b \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a+b+c & a+b+c & a+b+c \end{vmatrix}$$

$$= (a+b+c) \underbrace{\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}}_{\det = 0}$$

$$= 0$$



EXEMPLO



$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ \sin A & \sin B & \sin C \end{vmatrix} = 0$$

Lei Dos SENOS:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$a = 2R \cdot \sin A \quad ; \quad b = 2R \sin B \quad ; \quad c = 2R \cdot \sin C$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2R \cdot \sin A & 2R \cdot \sin B & 2R \cdot \sin C \\ \sin A & \sin B & \sin C \end{vmatrix} = 0$$

$$L_2 = 2R \cdot L_3 \text{ (LINHAS PROPORCIONAIS)}$$



REGRA DE CHIÓ

- * ABAIXAMENTO DE ORDEN DO DETERMINANTE
- * É NECESSÁRIO UM ELEMENTO "1".
- * É UM CASO PARTICULAR DO TEOREMA DE LAPLACE.

$$\begin{vmatrix} 1 & 2 & 4 & 1 \\ 3 & 3 & 1 & 2 \\ 5 & 3 & 2 & 3 \\ 1 & 4 & 2 & 1 \end{vmatrix} = (-1)^{2+3} \begin{vmatrix} 1-4.3 & 2-4.3 & 1-4.2 \\ 5-2.3 & 3-2.3 & 3-2.2 \\ 1-2.3 & 4-2.3 & 1-2.2 \end{vmatrix}$$

$$= (-1) \begin{vmatrix} -11 & -10 & -7 \\ -1 & -3 & -1 \\ -5 & -2 & -3 \end{vmatrix}$$



EXEMPLO

REDUZA O DETERMINANTE USANDO CHIÓ.

$$\begin{vmatrix} 2 & 4 & 1 & 3 \\ 3 & 0 & 1 & 3 \\ 3 & 2 & 3 & 1 \\ 1 & 4 & 2 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 4 & 1 & 3 \\ 3 & 0 & 1 & 3 \\ 3 & 2 & 3 & 1 \\ 1 & 4 & 2 & 2 \end{vmatrix} = (-1)^{4+1} \begin{vmatrix} 4-2.4 & 1-2.2 & 3-2.2 \\ 0-3.4 & 1-3.2 & 3-3.2 \\ 2-3.4 & 3-3.2 & 1-3.2 \end{vmatrix}$$

$$(-1) \begin{vmatrix} -4 & -3 & -1 \\ -12 & -5 & -3 \\ -10 & -3 & -5 \end{vmatrix}$$



EXEMPLO

CALCULE O DETERMINANTE

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 & 1 \\ 1 & 1 & 5 & 1 & 1 & 1 \\ 1 & 1 & 1 & 7 & 1 & 1 \\ 1 & 1 & 1 & 1 & 9 & 1 \\ 1 & 1 & 1 & 1 & 1 & 11 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 & 1 \\ 1 & 1 & 5 & 1 & 1 & 1 \\ 1 & 1 & 1 & 7 & 1 & 1 \\ 1 & 1 & 1 & 1 & 9 & 1 \\ 1 & 1 & 1 & 1 & 1 & 11 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{vmatrix}$$

$$= 2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 = 3840$$



CÁLCULO DA INVERSA POR ELEMENTOS

MOTIVADOR : CALCULE O ELEMENTO DA LINHA 2 E COLUNA 3 DA INVERSA DA MATRIZ:

$$\begin{vmatrix} 2 & 4 & 1 & 3 \\ 3 & 0 & 1 & 3 \\ 3 & 2 & 3 & 1 \\ 1 & 4 & 2 & 2 \end{vmatrix}$$

MATRICIALMENTE:

$$A^{-1} = \frac{1}{\det A} \cdot (A^*)^T$$

ELEMENTO A :
ELEMENTO

$$a_{ij}^{-1} = \frac{1}{\det A} \cdot a_{ji}^*$$



$$A = \begin{bmatrix} 4 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 4 & 2 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} & a_{12}^{-1} & a_{13}^{-1} \\ & & a_{23} \\ & & \end{bmatrix}$$

$\nearrow \frac{2}{5}$

$\rightarrow \det A = 10$

* CÁLCULO DE a_{12}^{-1}

$$\begin{bmatrix} 4 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 4 & 2 \end{bmatrix}$$

(The first column and the second row are crossed out in the original image)

$$a_{12}^{-1} = \frac{1}{\det A} \cdot a_{21}^{*}$$

$$= \frac{1}{10} \cdot (-1)^{2+1} \begin{vmatrix} 0 & 1 \\ 4 & 2 \end{vmatrix}$$

$$= \frac{1}{10} \cdot (-1) (0 - 4)$$

$$= \frac{4}{10} = \frac{2}{5}$$



* CÁLCULO DE a_{13}^{-1}

$$\begin{bmatrix} 4 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 4 & 2 \end{bmatrix}$$

$$a_{13}^{-1} = \frac{1}{\det A} \cdot a_{31}^*$$

$$= \frac{1}{10} \cdot (-1)^{2+1} \cdot \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix}$$

$$= \frac{1}{10} \cdot (-1) (0 - 2)$$

$$= \frac{-2}{10} = -\frac{1}{5}$$

* CÁLCULO DE a_{23}^{-1}

$$\begin{bmatrix} 4 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 4 & 2 \end{bmatrix}$$

$$a_{23}^{-1} = \frac{1}{\det A} \cdot a_{32}^*$$

$$= \frac{1}{10} \cdot (-1)^{2+3} \cdot \begin{vmatrix} 4 & 1 \\ 3 & 1 \end{vmatrix}$$

$$= \frac{1}{10} \cdot (-1) (4 - 3)$$

$$= \frac{1}{10} \cdot (-1) (1) = -\frac{1}{10}$$



EXEMPLO

CALCULE O ELEMENTO DA LINHA 3 E COLUNA 2 DA INVERSA DA MATRIZ ABAIXO.

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 5 & 3 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 1 & 3 & 0 & 1 & 3 \\ 1 & 5 & 3 & 1 & 5 \\ 2 & 2 & 1 & 2 & 2 \end{vmatrix}$$

$$= (5 + 18 + 0) - (6 + 3)$$

$$\det A = 14$$



ELEMENTO a_{32}^{-1} :

$$\begin{bmatrix} 1 & 3 & 0 \\ 1 & 5 & 3 \\ 2 & 2 & 1 \end{bmatrix}$$

$$a_{32}^{-1} = \frac{1}{\det A} \cdot a_{23}^*$$

$$= \frac{1}{14} \cdot (-1)^{2+3} \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix}$$

$$= \frac{1}{14} (-1)(2 - 6)$$

$$= \frac{4}{14}$$

$$a_{32}^{-1} = \frac{2}{7}$$

