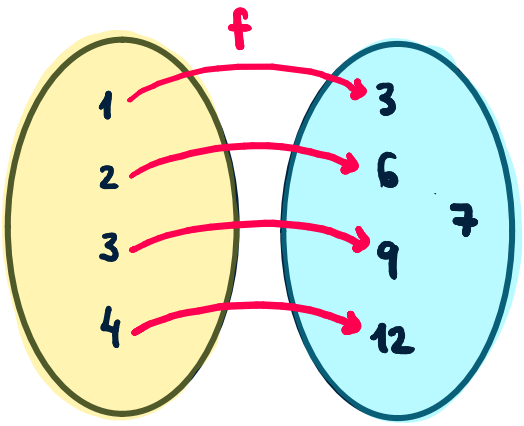
FUNÇÕES INVERSAS

· INTRODUÇÃO ·

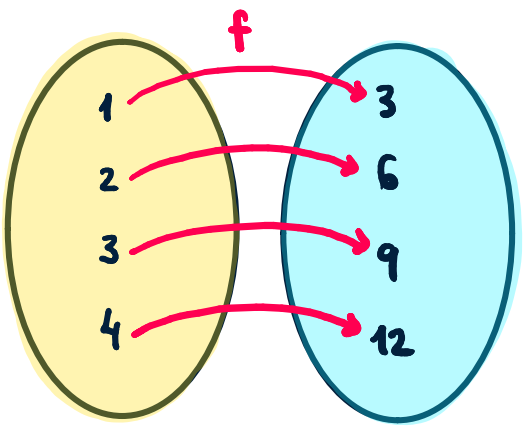
A função inversa f^{-1} só existe se:

(i) f é sobrejetora:

✗ ~~NÃO SOBREJETORA~~



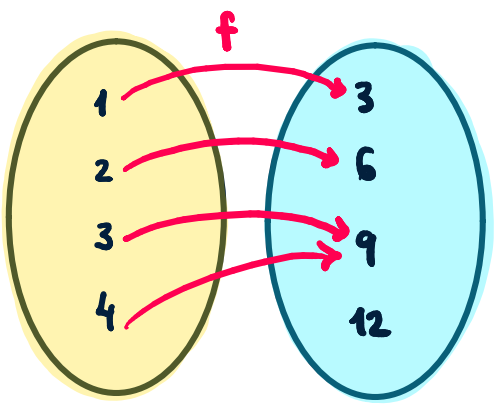
✓ ~~SOBREJETORA~~



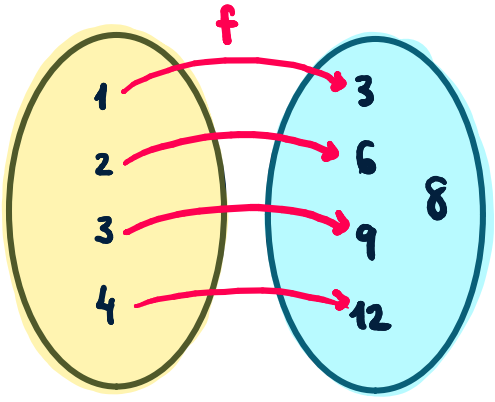
$Im(f) = C.D.(f)$

(ii) f é injetora:

✗ ~~NÃO INJETORA~~

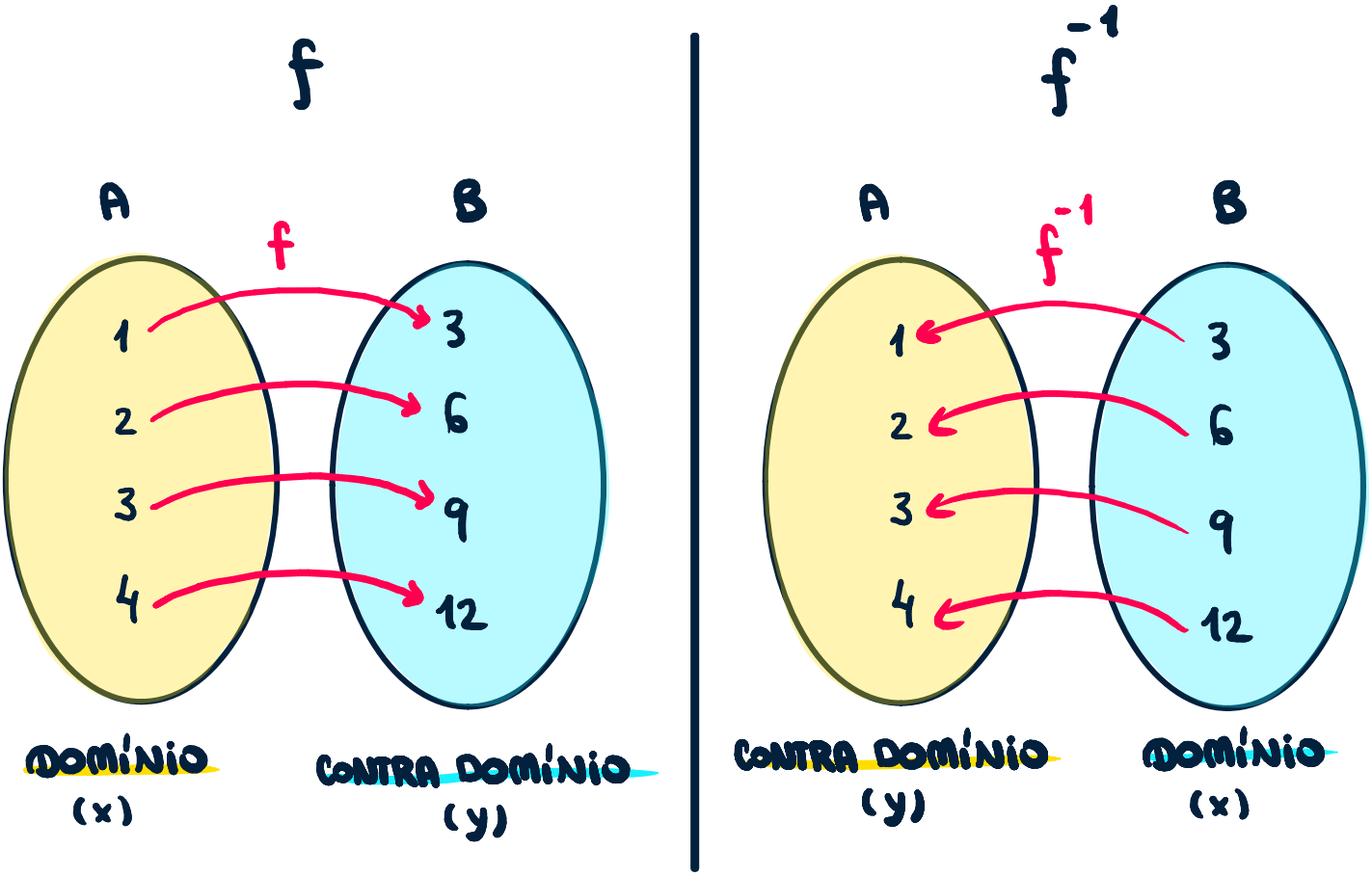


✓ ~~INJETORA~~

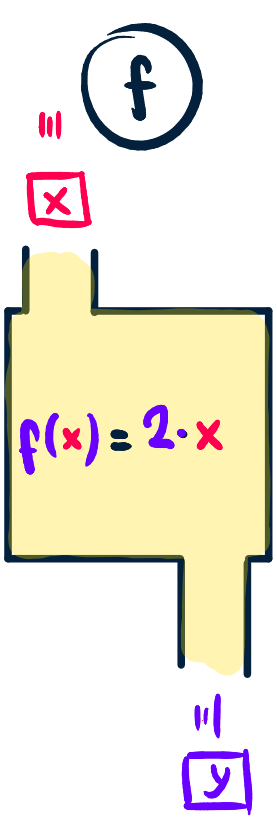


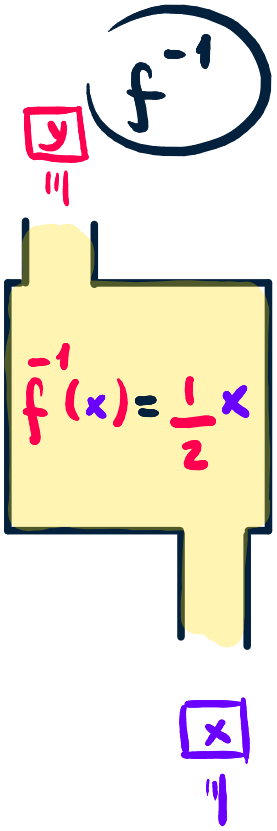
1.1 · VISUALIZANDO UMA FUNÇÃO INVERSA

a) DIAGRAMA DE FLECHAS

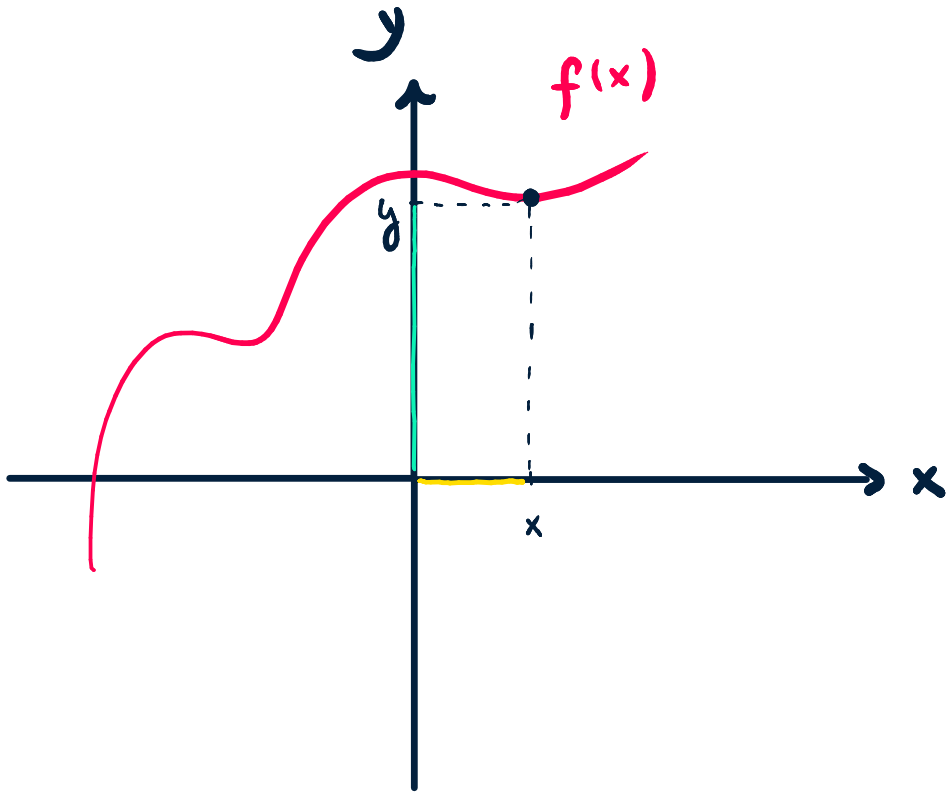


b) MÁQUINA

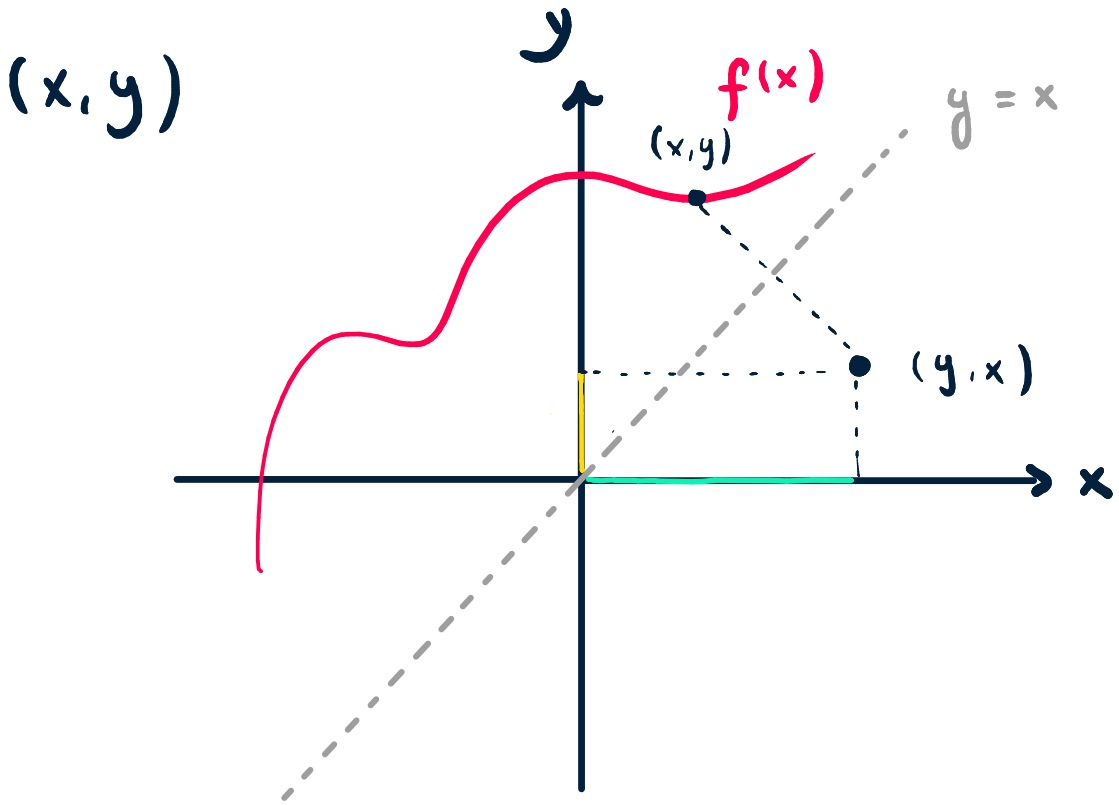




c) GRÁFICO



Se $(x,y) \in f \rightarrow (y,x) \in f^{-1}$



FUNÇÕES TRIGONOMÉTRICAS INVERSAS

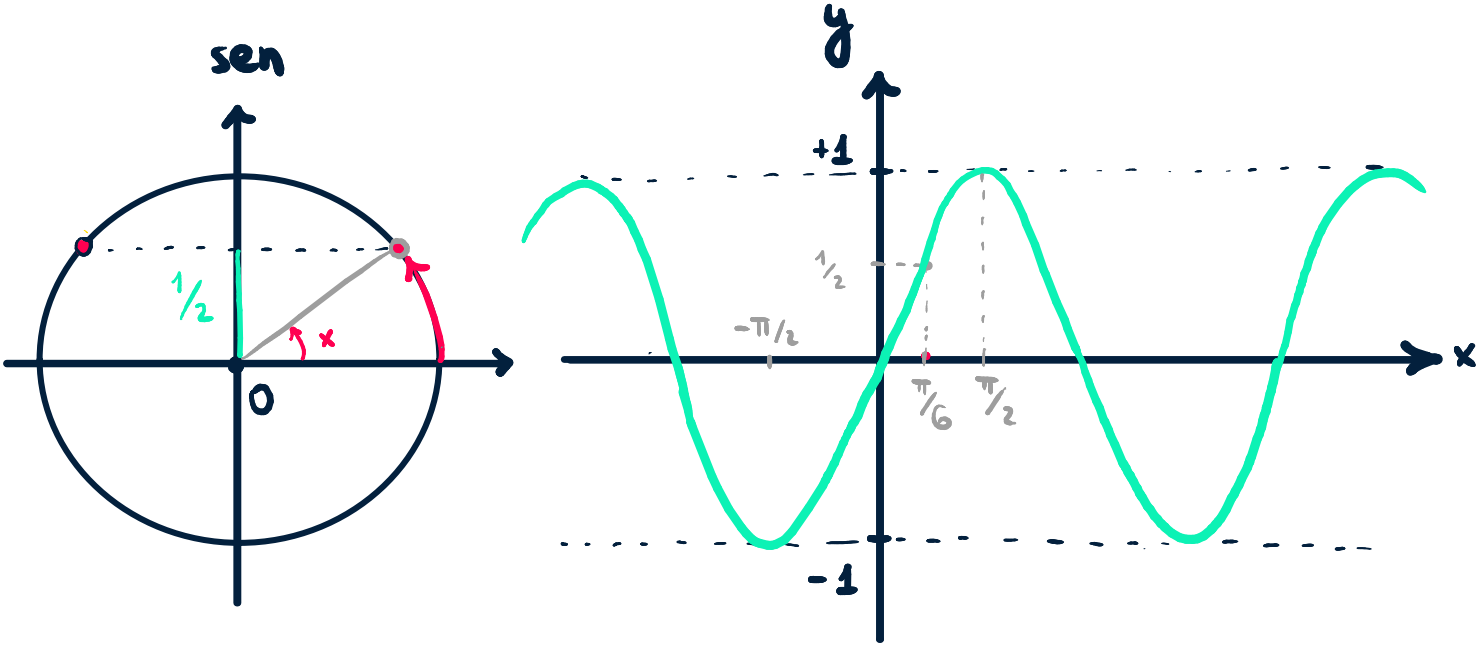
• INVERSA DO SENO : ARCO-SENO •

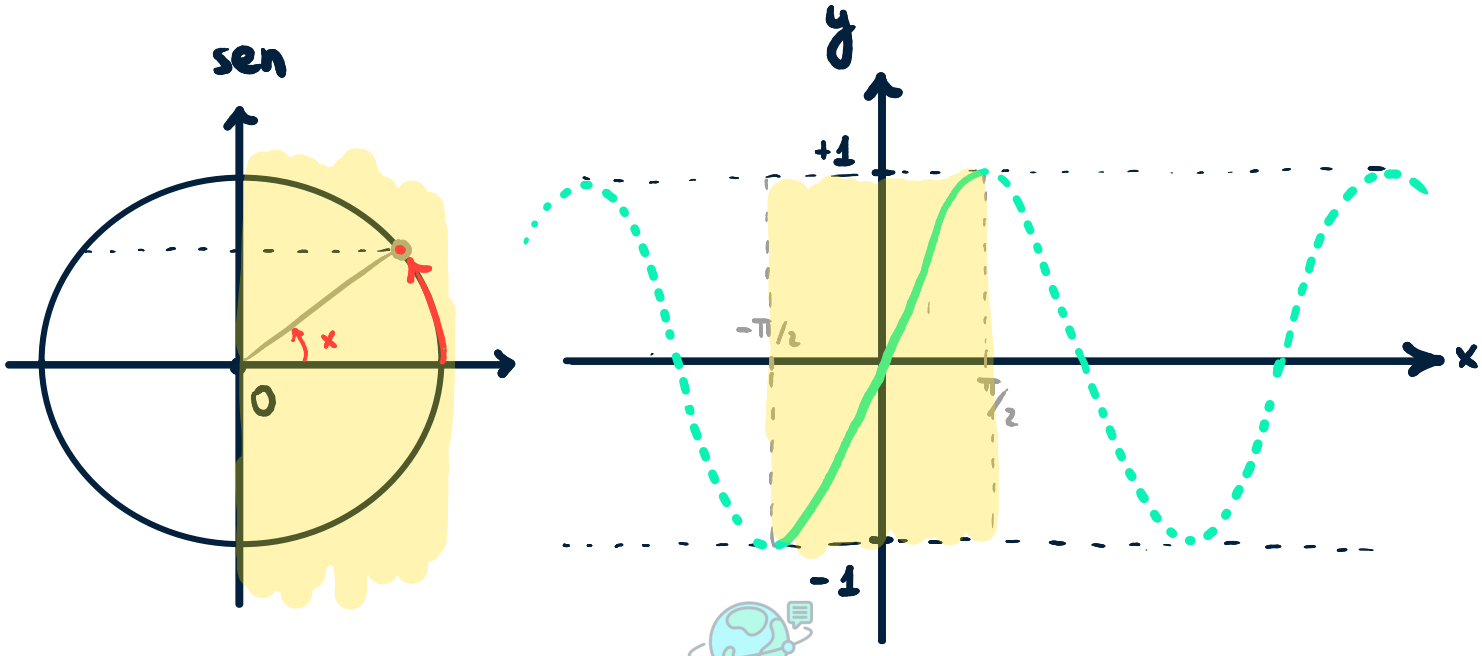
Seja a função :

$f(x) = \text{Sen}(x)$

$D(f) = [-\pi/2 , \pi/2]$

$Im(f) = [-1, 1]$





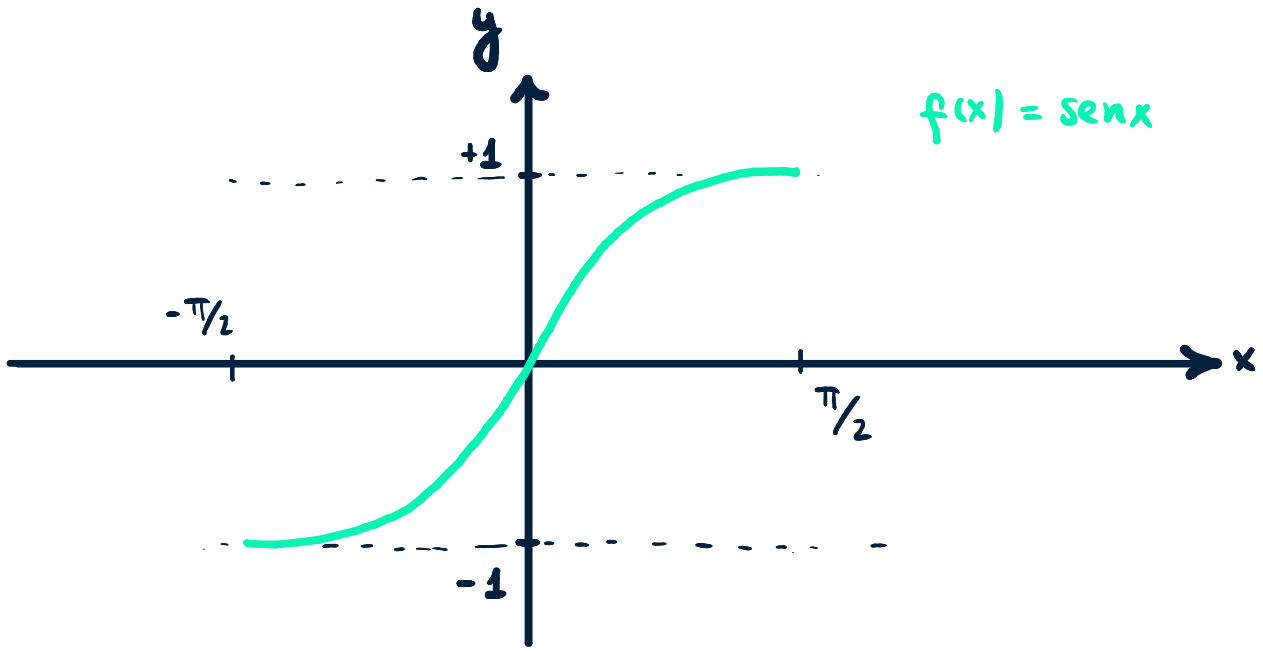
Logo, tal função é bijetora.

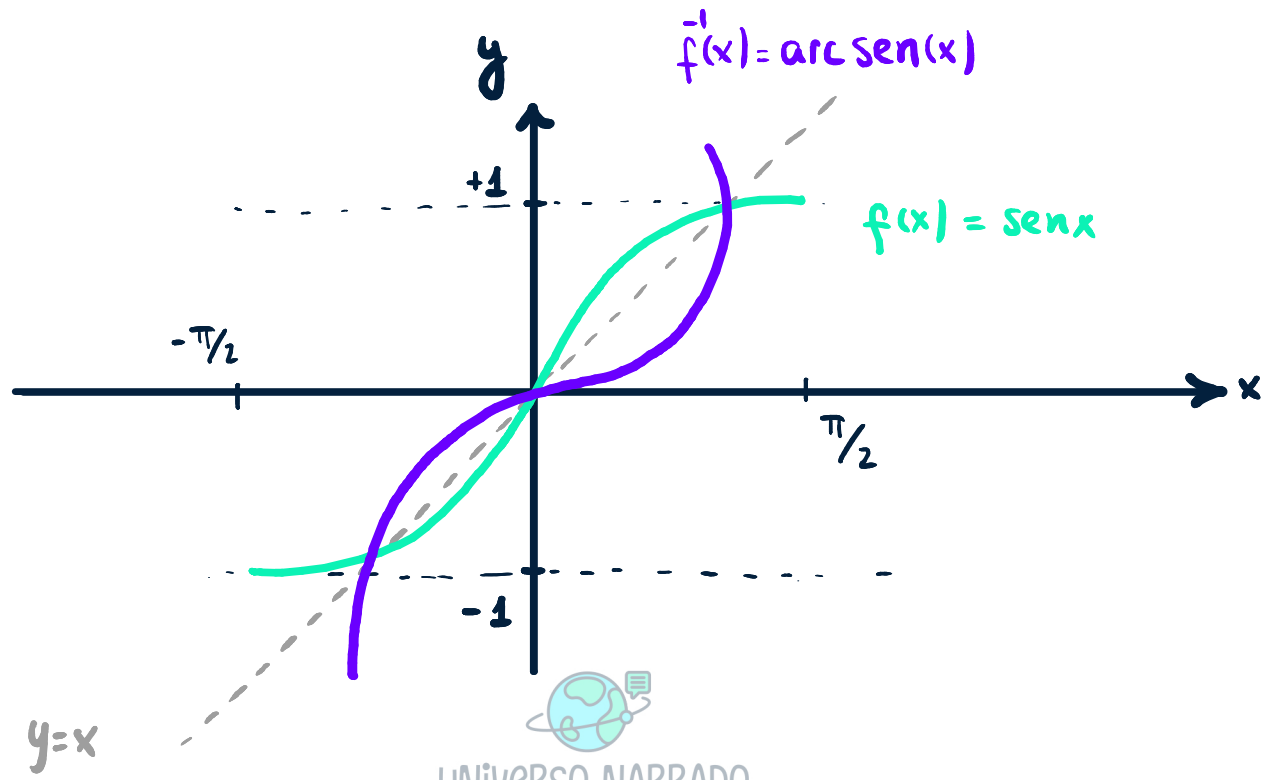
Sua inversa é:

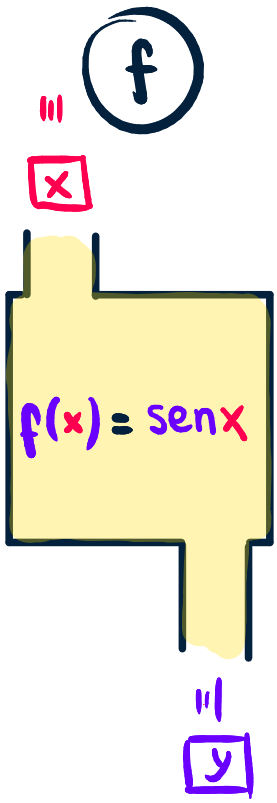
$$f^{-1}(x) = \text{sen}^{-1}(x) = \text{arc sen}(x)$$

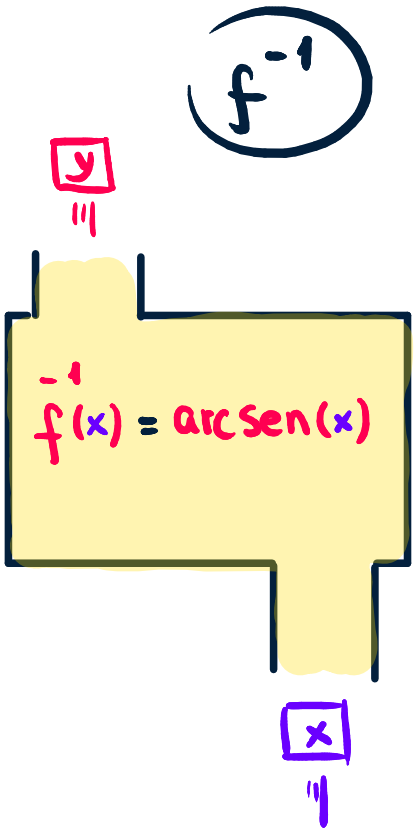
$$D(f^{-1}) = [-1, 1]$$

$$Im(f^{-1}) = [-\pi/2, \pi/2]$$









- arc sen (0) = 0
- arc sen (1/2) = 30°
- arc sen ($\sqrt{2}$/2) = 45°
- arc sen ($\sqrt{3}$/2) = 60°

EXEMPLO

Determine o domínio e imagem da função

$f(x) = \text{arc sen}(3x + 2)$

R.:

Imagem: $-\pi/2 \leq \text{arc sen}(\blacksquare) \leq +\pi/2$

$$Im(f) = [-\pi/2, \pi/2]$$

Domínio: $y = \text{arc sen}(3x+2)$

$$\text{sen } y = 3x + 2$$

$$-1 \leq \text{sen } y \leq +1$$

$$-1 \leq 3x + 2 \leq +1$$

$$-3 \leq 3x \leq -1$$

$$-1 \leq x \leq -1/3$$

$$D(f) = x \in [-1, -1/3]$$

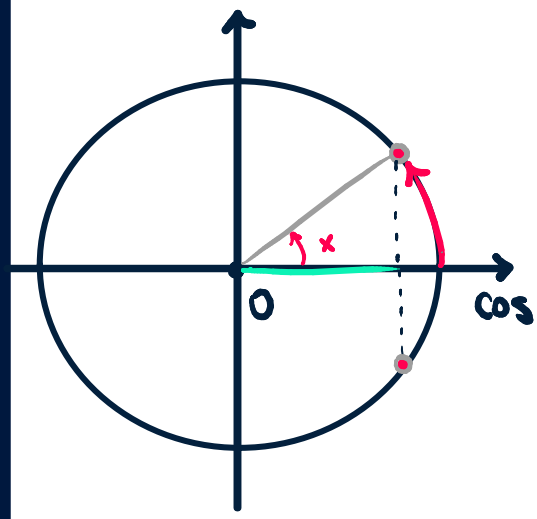
• INVERSA DO COSSENO : ARCO-COSSENO •

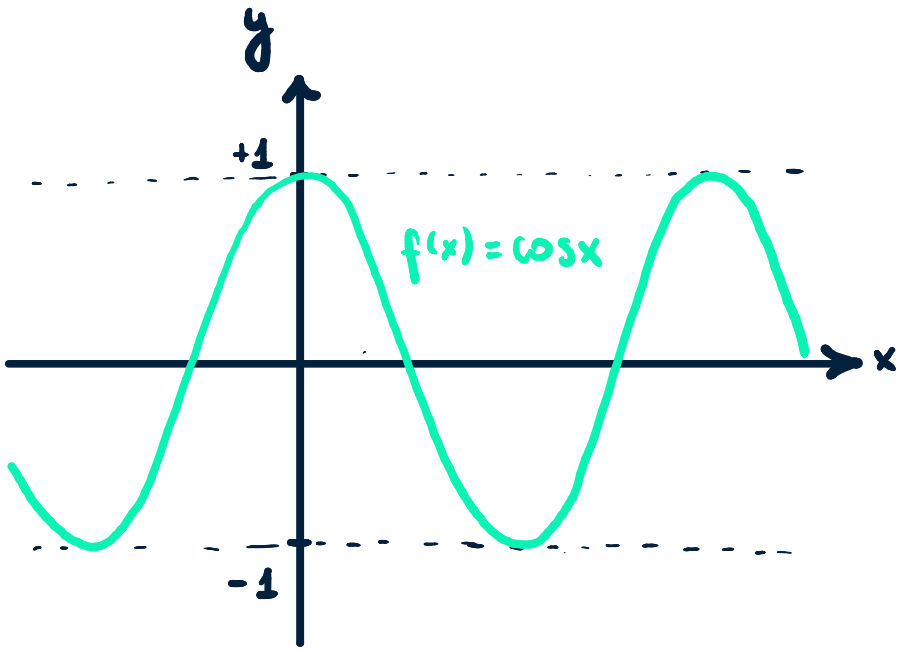
Seja a função :

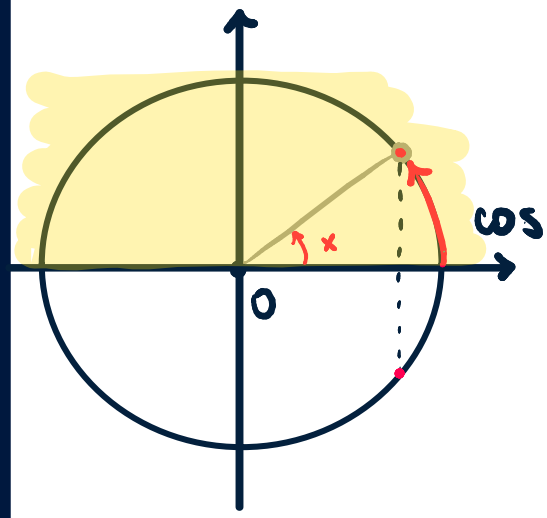
$$f(x) = \cos(x)$$

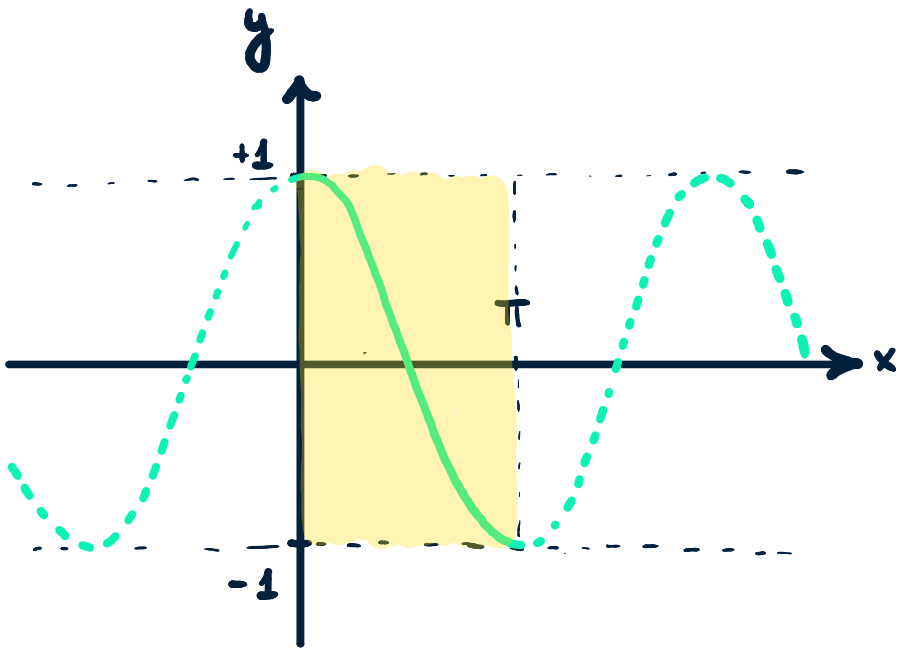
$$D(f) = [0, \pi]$$

$$Im(f) = [-1, 1]$$









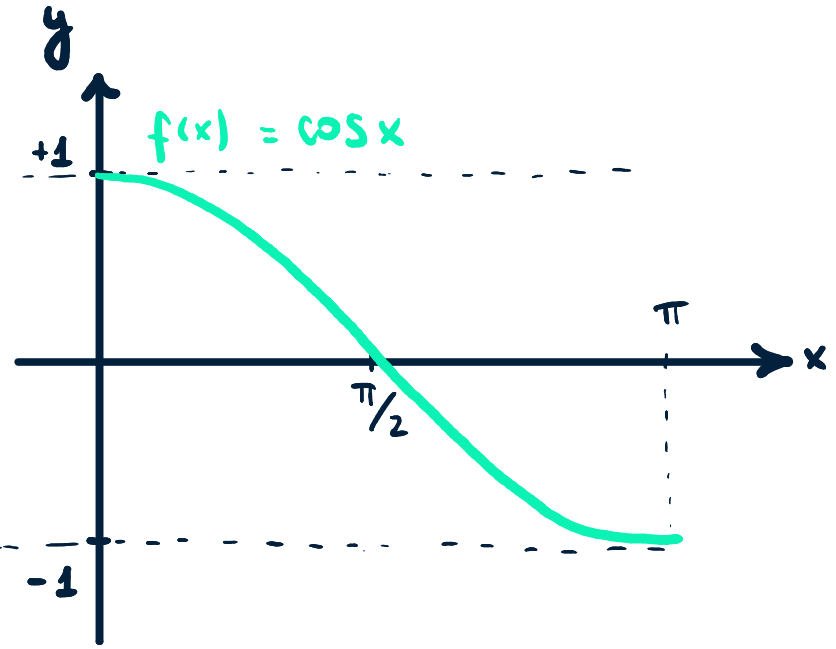
Logo, tal função é bijetora.

Sua inversa é:

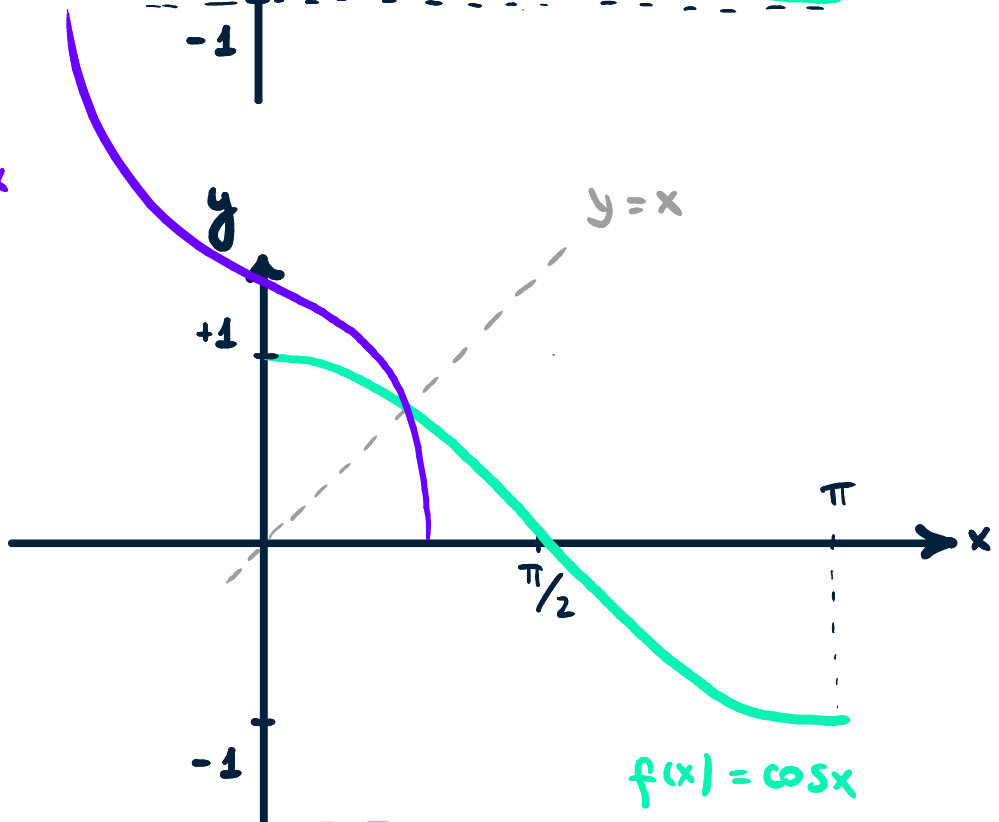
$$f^{-1}(x) = \cos^{-1}(x) = \arccos(x)$$

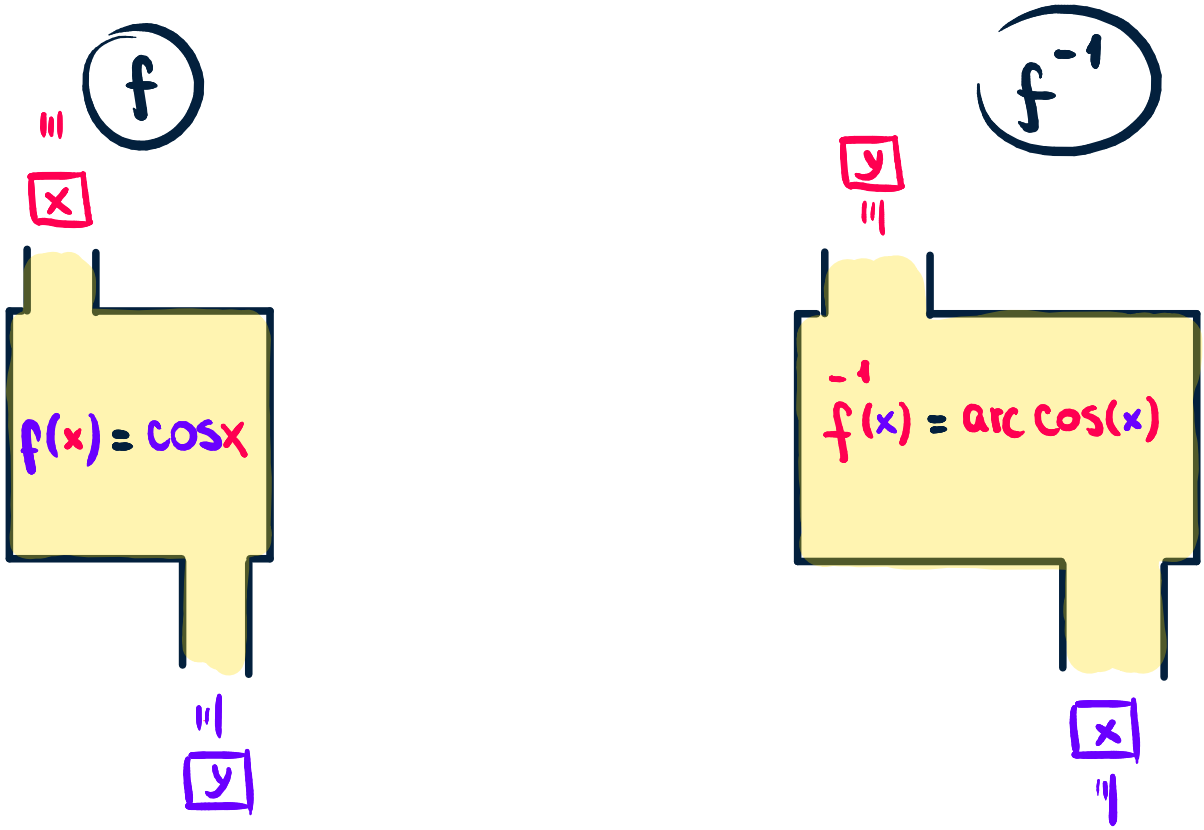
$$D(f^{-1}) = [-1, 1]$$

$$Im(f^{-1}) = [0, \pi]$$



$f^{-1}(x) = \arccos x$





- arc cos(0) = $\pi/2$
- arc cos(1/2) = $\pi/3$
- arc sen($-\sqrt{2}/2$) = $3\pi/4$
- arc cos($-\sqrt{3}/2$) = $5\pi/6$

EXEMPLO

Determine o domínio e imagem da função

$$f(x) = 2\arccos(x^2 + 1) + \pi$$

R.:

. **Domínio:** $D(f) = x \in \mathbb{R} \mid x = 0$

. **Imagem:** $f(0) = 2\arccos(0^2 + 1) + \pi$

$$= 2 \cdot \underbrace{\arccos(1)}_{0} + \pi = \pi$$

$$Im(f) = y \in \mathbb{R} \mid y = \pi$$

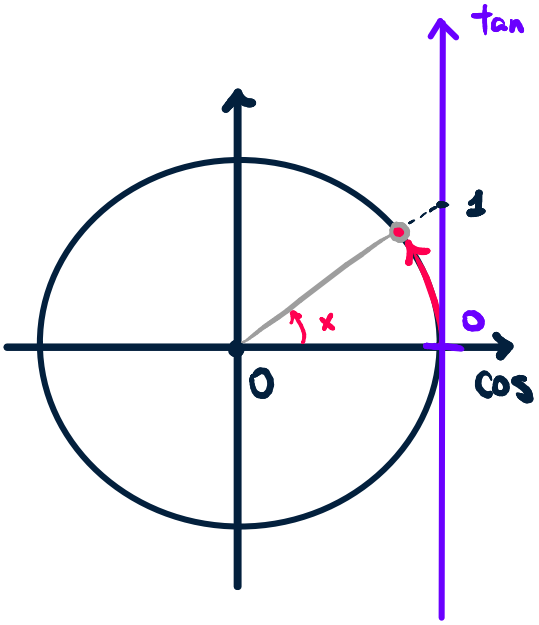
• INVERSA DA TANGENTE: ARCO-TANGENTE •

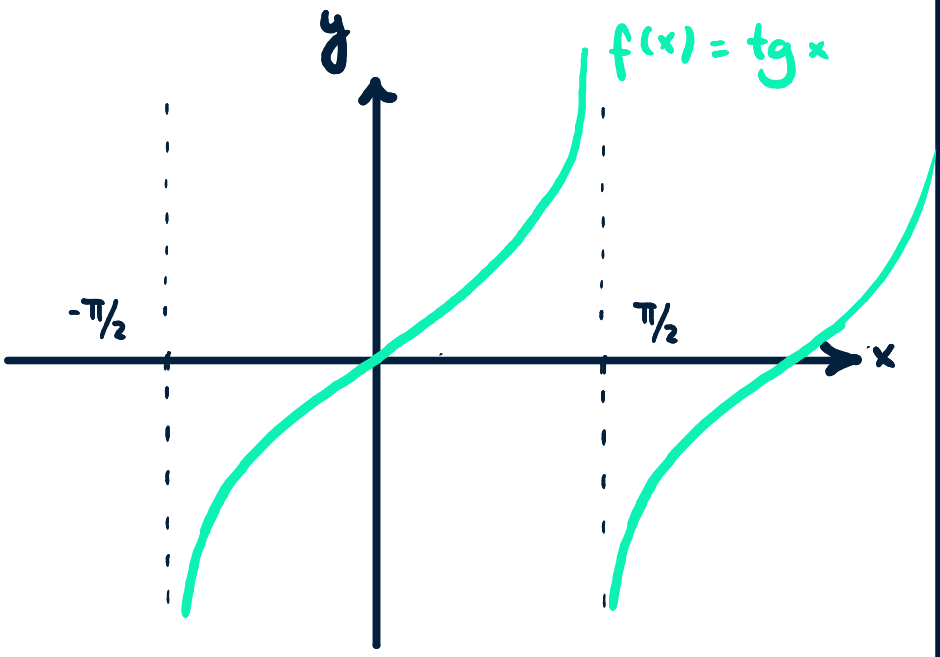
Seja a função :

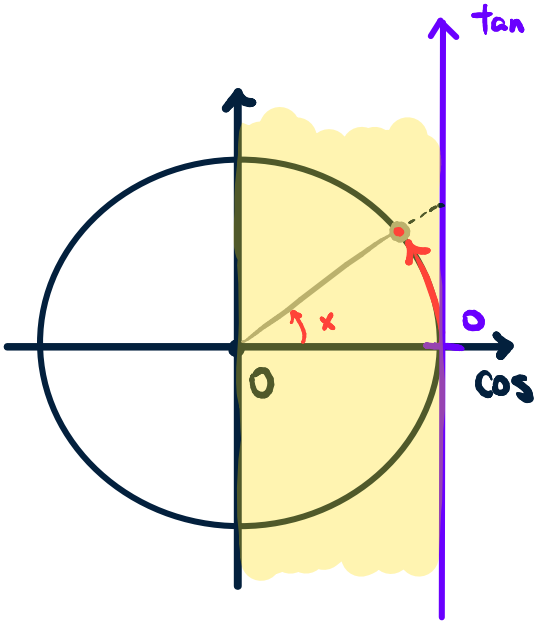
$$f(x) = tg(x)$$

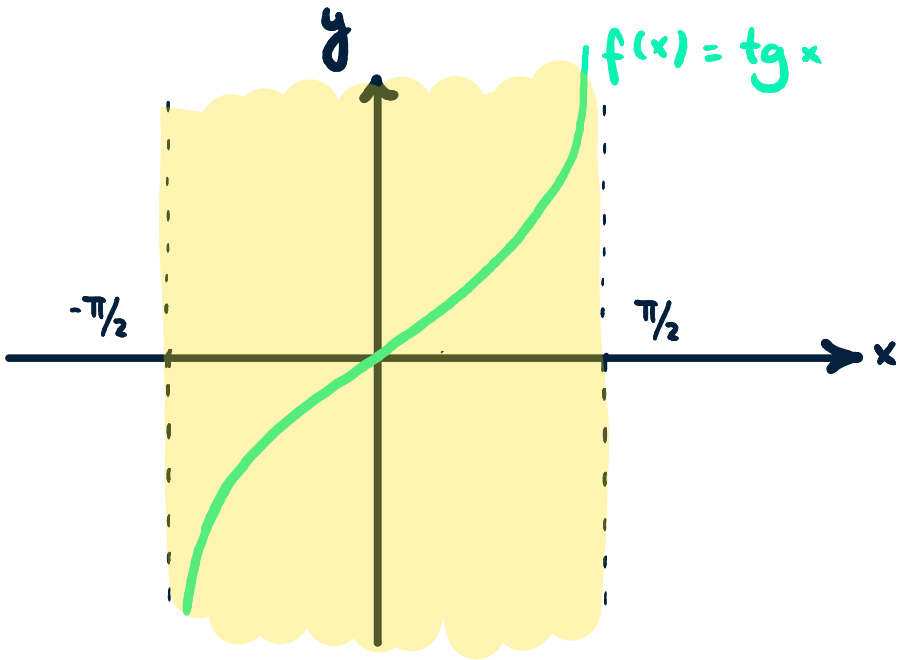
$$D(f) = [-\pi/2 , \pi/2]$$

$$Im(f) = \mathbb{R}$$









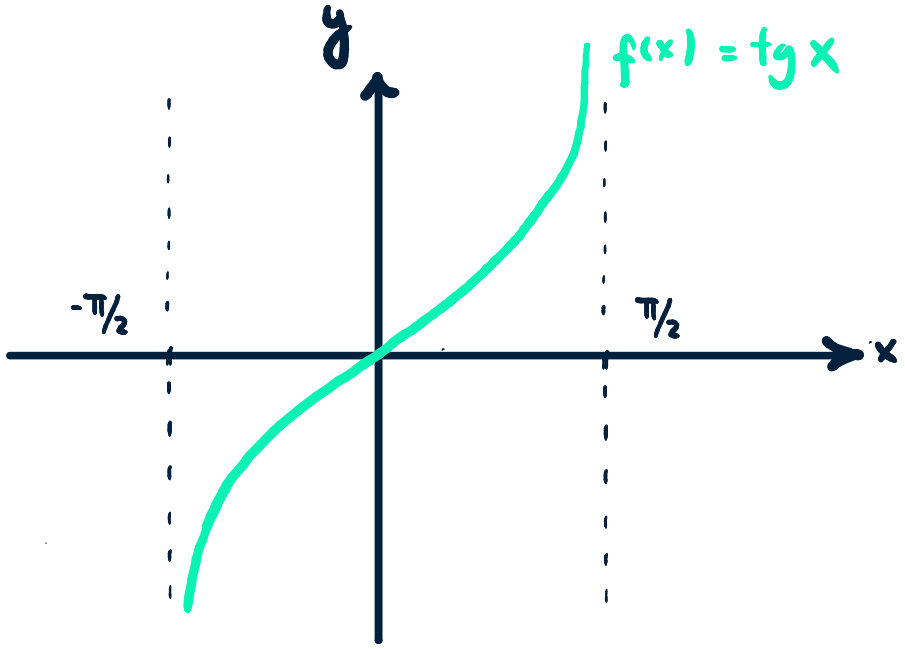
Logo, tal função é bijetora.

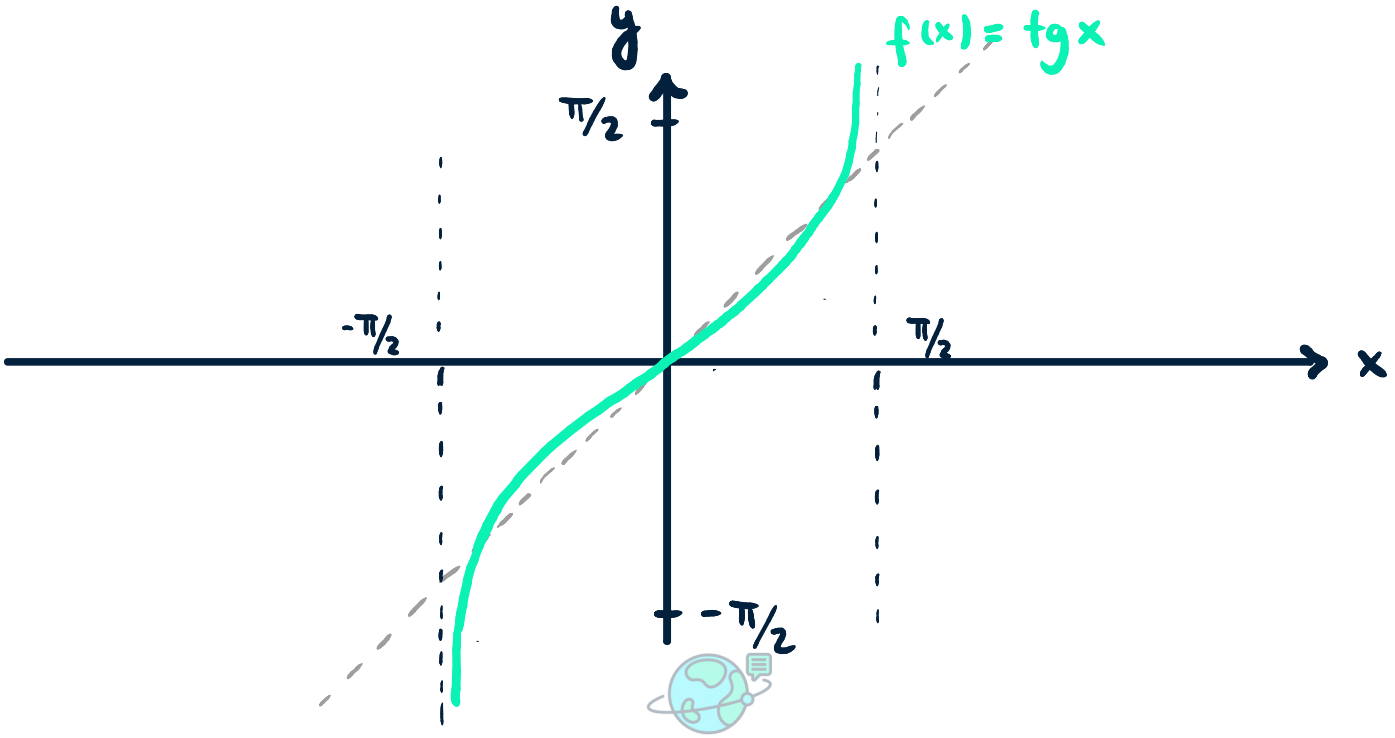
Sua inversa é:

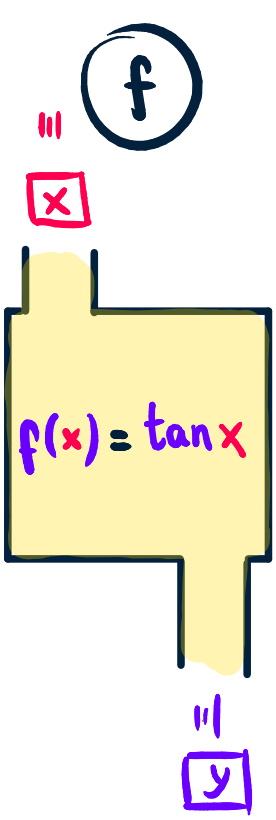
$$f^{-1}(x) = tg^{-1}(x) = \text{arc}\,tg(x)$$

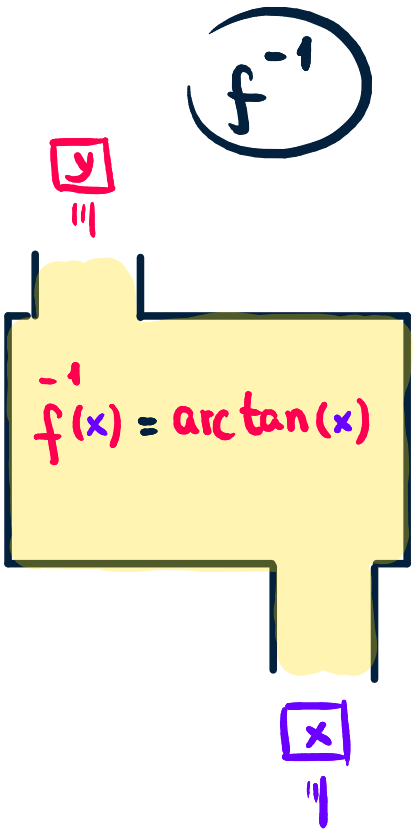
$$D(f^{-1}) = \mathbb{R}$$

$$Im(f^{-1}) = [-\pi/2, \pi/2]$$









- arc tg (0) = 0

- arc tg (-1) = $-\pi/4$

- arc tg ($\sqrt{3}$) = $\pi/3$

EXEMPLO

Determine o domínio e imagem da função

$$g(x) = \frac{4}{5} \operatorname{arc\,tg}(x+1)$$

R.:

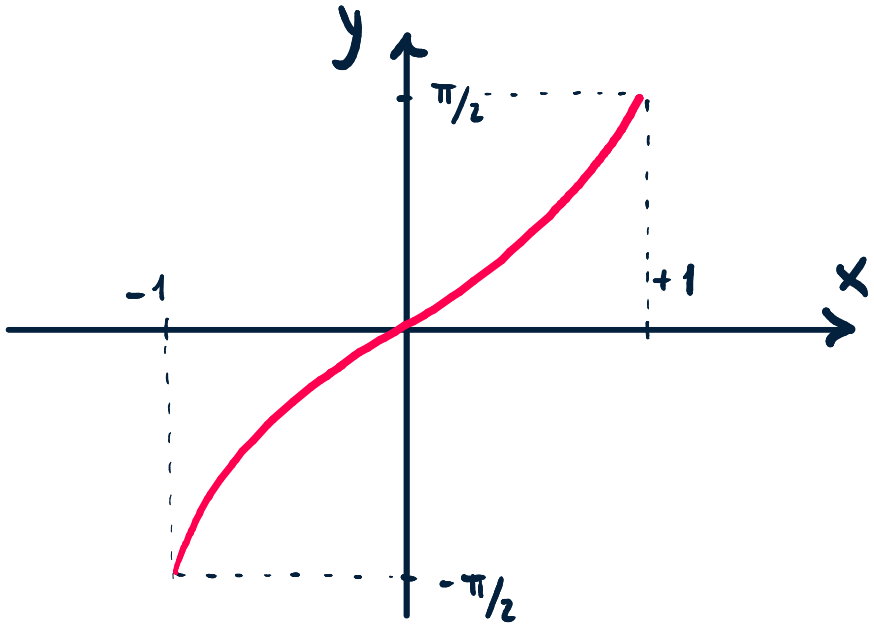
. **Domínio** : $D(f): x \in \mathbb{R}$

. **Imagem** : $Im(f): y \in \mathbb{R} \mid y \in [-2\pi/5 , 2\pi/5]$

$$-\frac{\pi}{2}\cdot\frac{4}{5} \leq \operatorname{arc\,tg}(\sim)\cdot\frac{4}{5} \leq \frac{\pi}{2}\cdot\frac{4}{5}$$

$$\frac{-2\pi}{5} \leq g(x) \leq \frac{2\pi}{5}$$

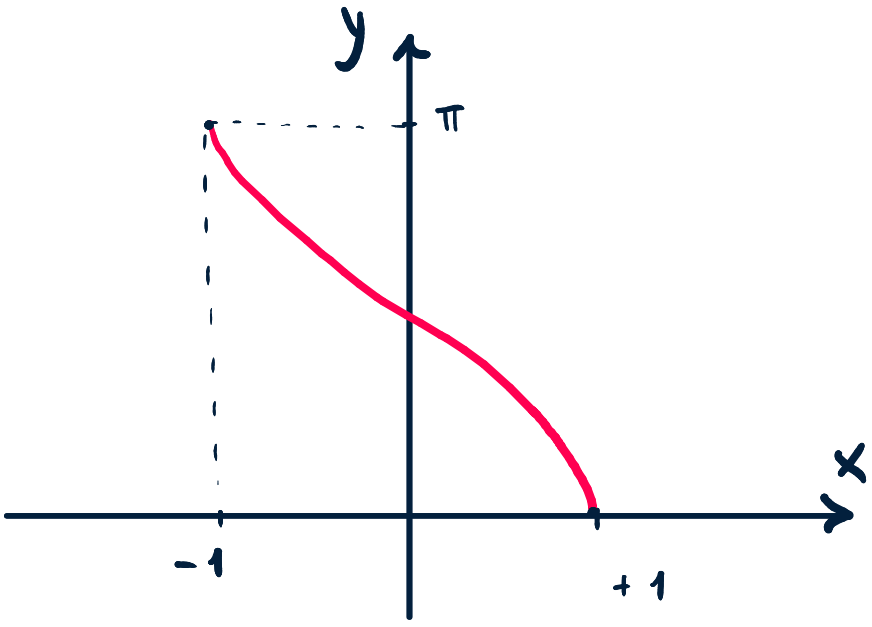
RESUMO



$f^{-1}(x) = sen^{-1}(x) = arc\,sen(x)$

$D(f^{-1}) = [-1, 1]$

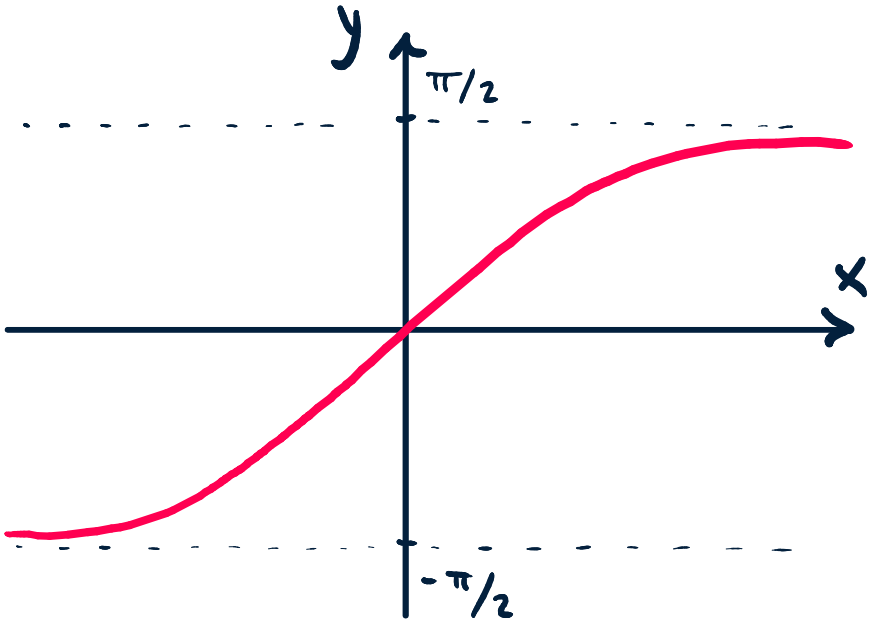
$Im(f^{-1}) = [-\pi/2, \pi/2]$



$f^{-1}(x) = cos^{-1}(x) = arc\,cos(x)$

$D(f^{-1}) = [-1, 1]$

$Im(f^{-1}) = [0, \pi]$



$f^{-1}(x) = tg^{-1}(x) = \text{arc}\,tg(x)$

$D(f^{-1}) = \mathbb{R}$

$Im(f^{-1}) = [-\pi/2, \pi/2]$