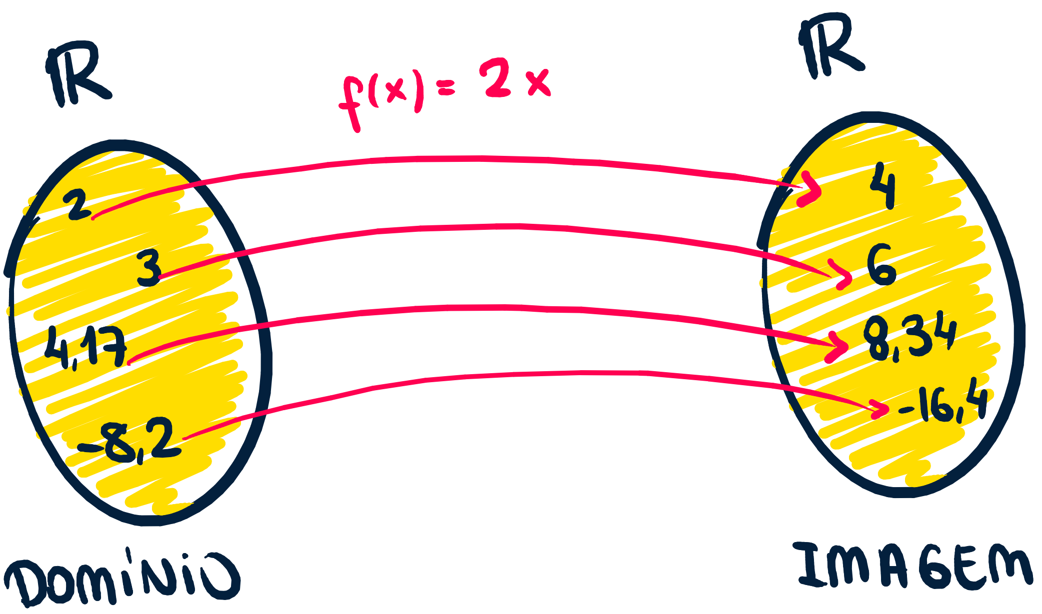
·FUNÇÃO AFIM·

DOMÍNIO IMAGEM

DEFINIÇÃO

Uma função $f : \mathbb{R} \to \mathbb{R}$ é dita afim quando existem constantes $a, b \in \mathbb{R}$ tais que $f(x) = ax + b$, para todo $x \in \mathbb{R}$



· EXEMPLOS ·

- $f(x) = \overbrace{2}^{a}x + \overbrace{1}^{b}$ ✓

- $g(x) = \overbrace{-5}^{a}x + \overbrace{17}^{b}$ ✓

- $h(x) = \overbrace{3}^{a}x$ $\quad b = 0$ ✓

- $p(x) = 2x + x^2$ ✗

- $t(x) = \sqrt{x - 1}$ ✗

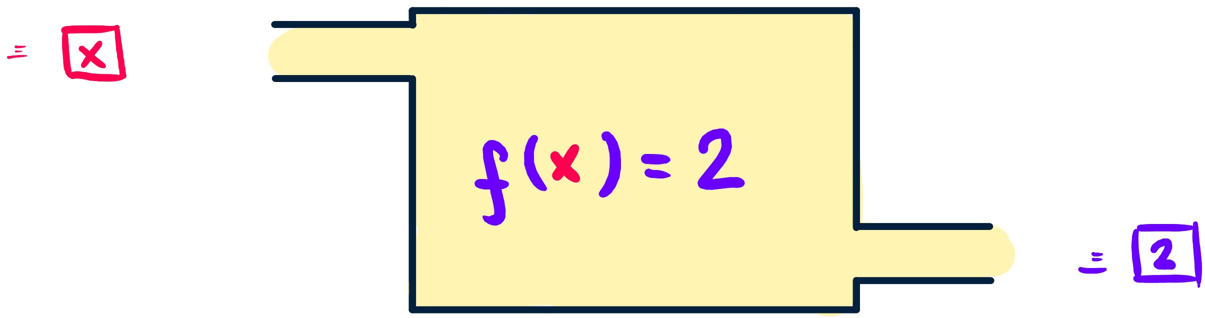
- $m(x) = \overbrace{}^{a=1}x - \overbrace{1}^{b=-1}$ ✓

· FUNÇÃO CONSTANTE ·

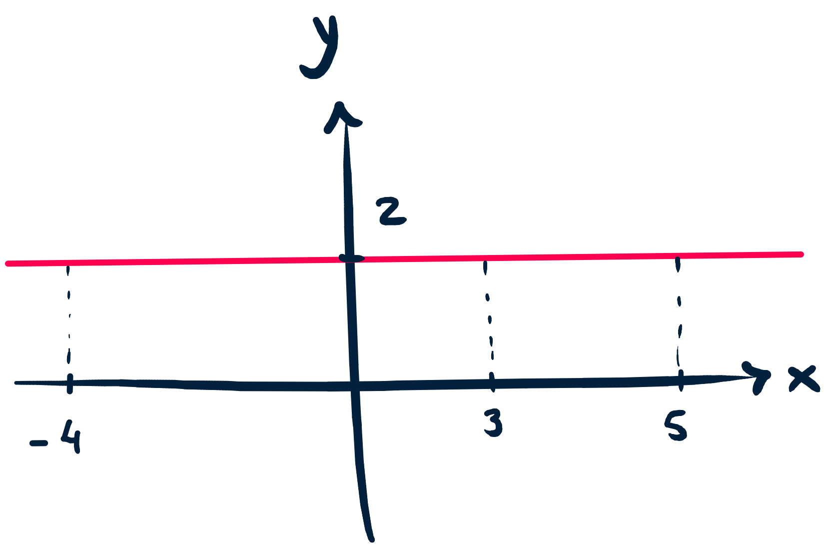
$$f(x) = \underbrace{a}_{\text{zero}}x + b = b \quad \therefore \quad f(x) = b$$

EXEMPLO

$$f(x) = 2$$



x	f(x)
-1	2
3	2
-4	2
9	2

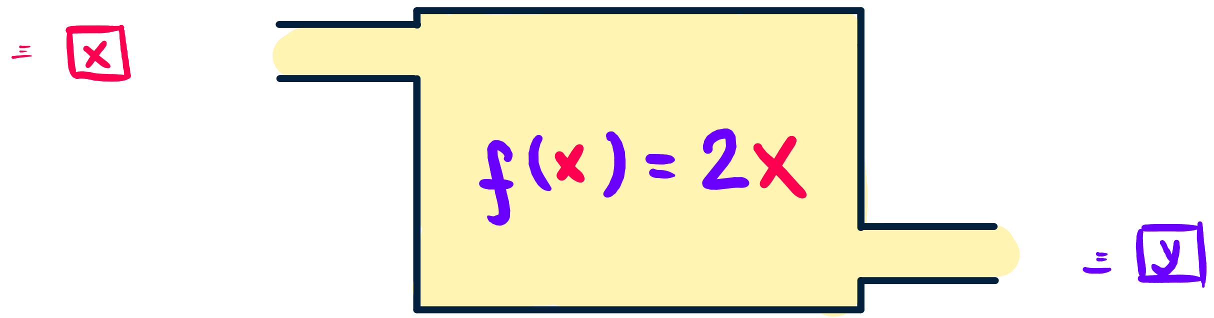


• FUNÇÃO LINEAR:

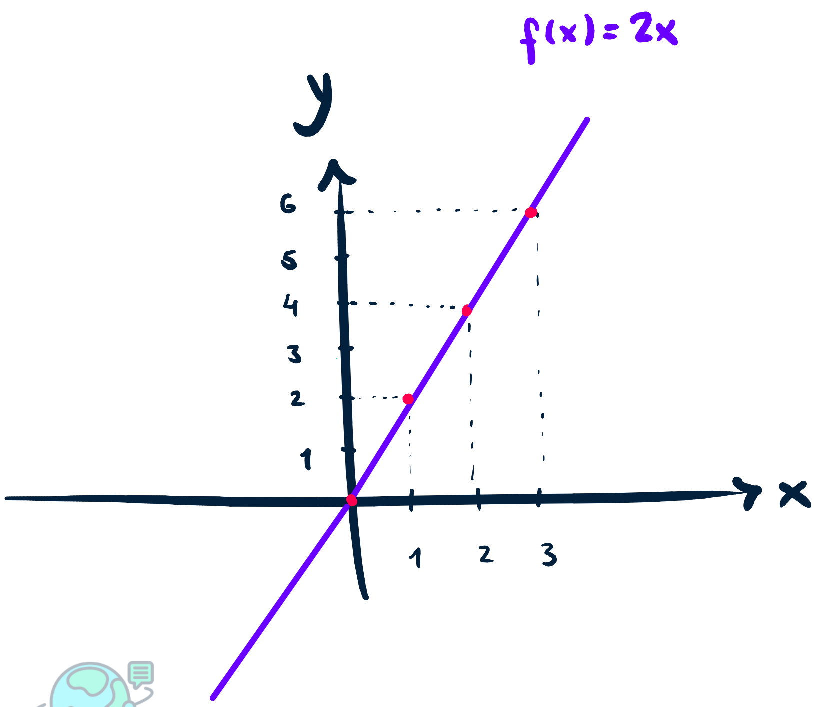
$$f(x) = ax + \underbrace{b}_{\text{zero}} = a \cdot x \quad \therefore \quad f(x) = a \cdot x$$

EXEMPLO

$$f(x) = 2x$$



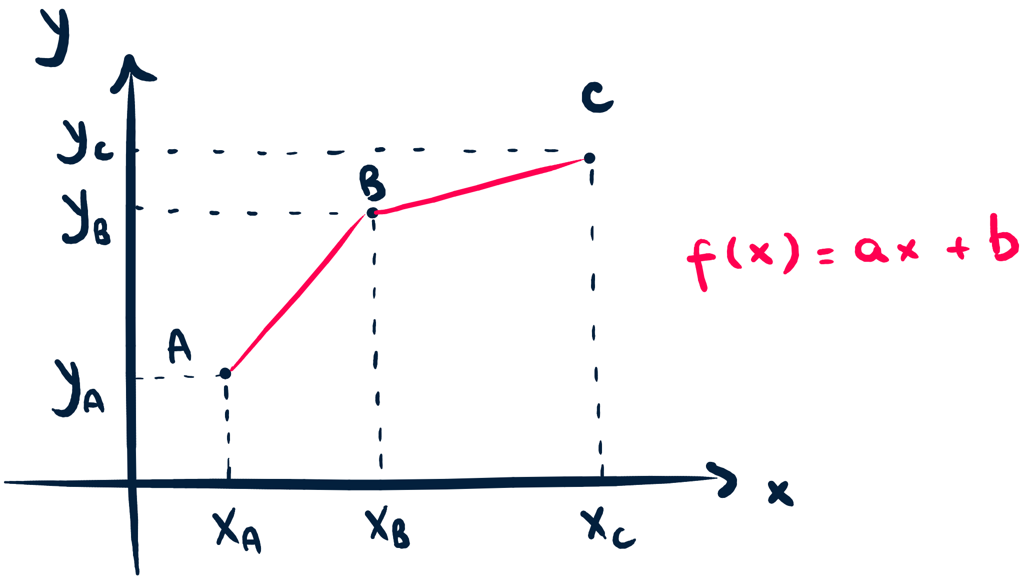
x	f(x)
0	0
1	2
2	4
3	6



TEOREMA

O gráfico de uma função afim é uma linha reta.

. DEMONSTRAÇÃO .



$A \in f$: $y_A = a x_A + b$ (I)

$B \in f$: $y_B = a x_B + b$ (II)

$C \in f$: $y_C = a x_C + b$ (III)

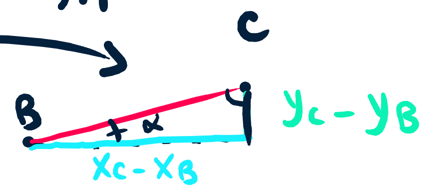
$$(III) - (II): \quad y_C - y_B = a\,(x_C - x_B)$$

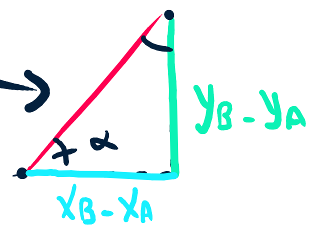
$\div$

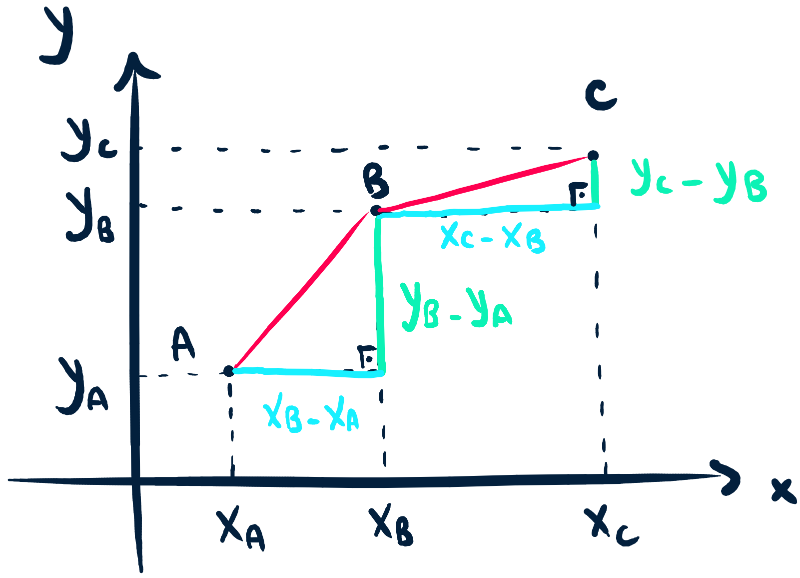
$$(II) - (I): \quad y_B - y_A = a\,(x_B - x_A)$$

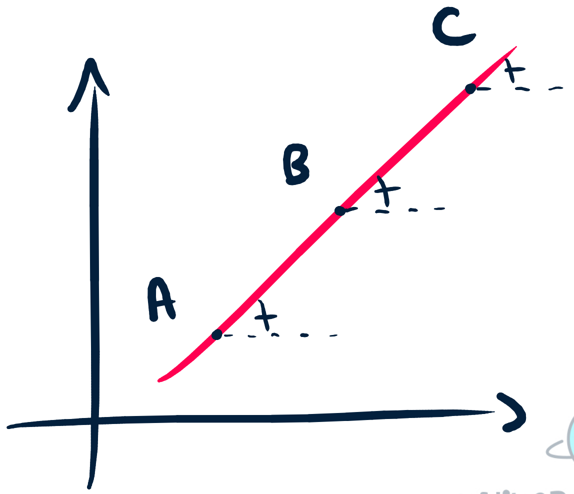
SEMELHANÇA

$$\frac{y_C - y_B}{y_B - y_A} = \frac{(x_C - x_B)}{(x_B - x_A)}$$









· FUNÇÃO AFIM ·

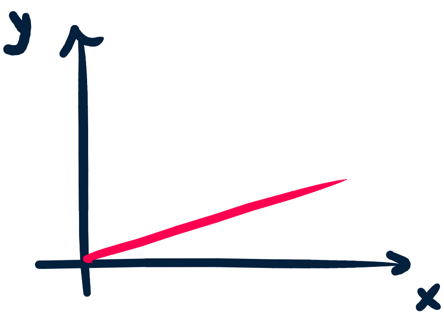
COEFICIENTES

$$f(x) = a \cdot x + b$$

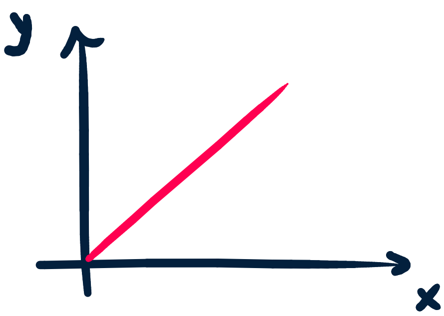
(i) COEFICIENTE ANGULAR (a)

· MEDE A INCLINAÇÃO DA RETA (EM RELAÇÃO À HORIZONTAL)

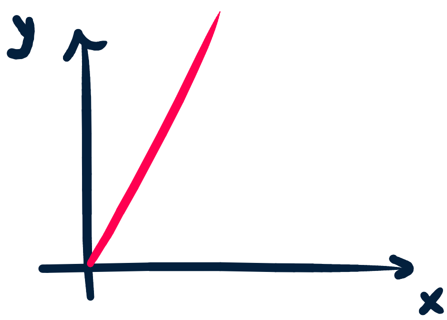
$$f(x) = a \cdot x$$



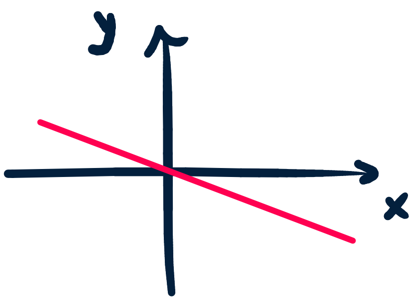
$a = 0,3$



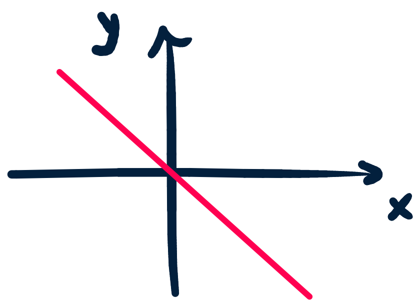
$a = 1$



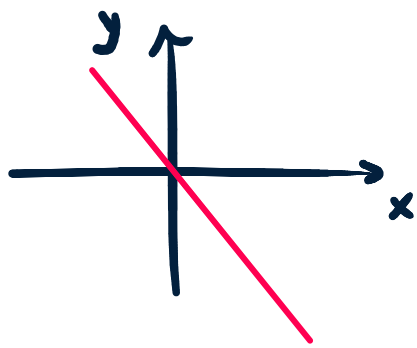
$a = 3$



$a = -0,3$



$a = -1$



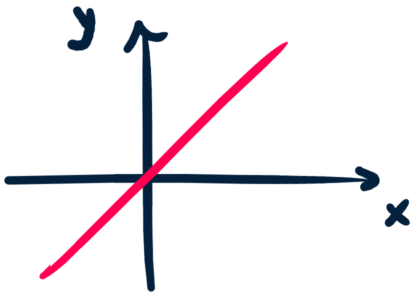
$a = -3$

(ii) COEFICIENTE LINEAR (b)

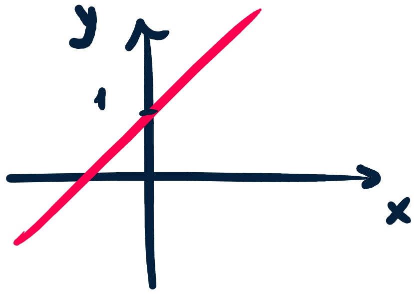
- MEDE ONDE A RETA INTERCEPTA O EIXO Y

$$f(x) = 1\cdot x + b \quad \therefore \quad f(0) = 1\cdot 0 + b = b$$

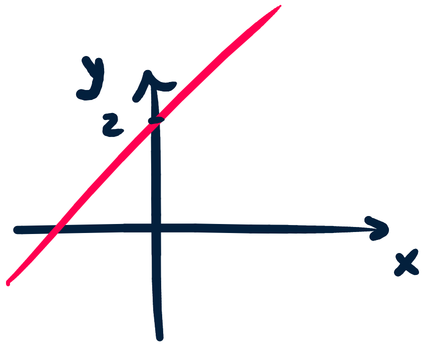
VALOR DE $f(0) = b$



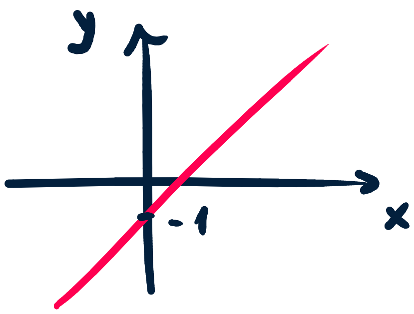
$b = 0$



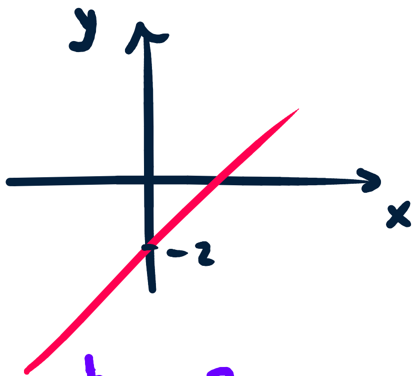
$b = 1$



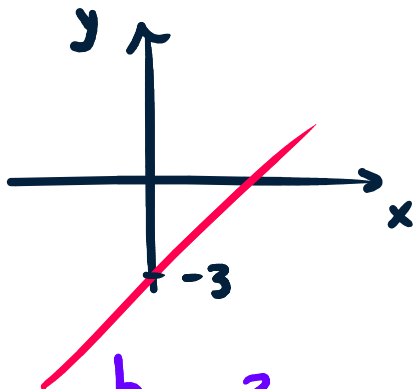
$b = 2$



$b = -1$

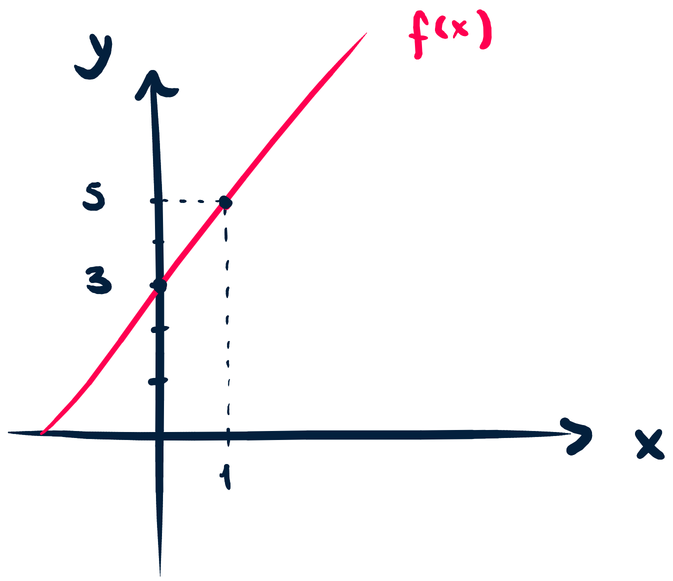


$b = -2$

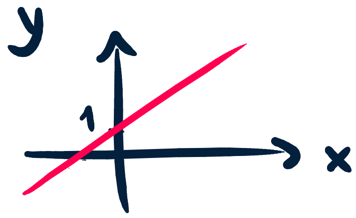


$b = -3$

$$f(x) = 2x + 3$$

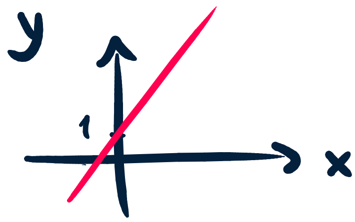


• FUNÇÃO AFIM : CRESCENTE VS DECRESCENTE

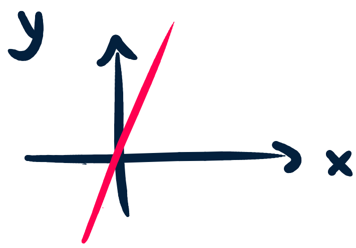


$f(x) = x + 1$

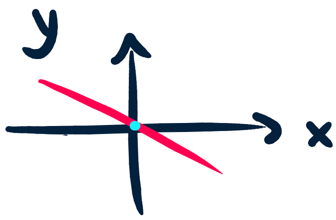
CRESCENTE : $a(+)$



$f(x) = 3x + 1$

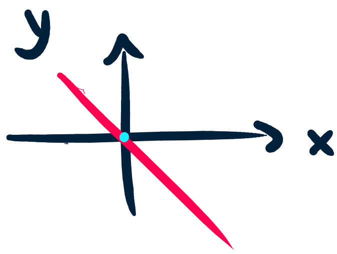


$f(x) = 5x$

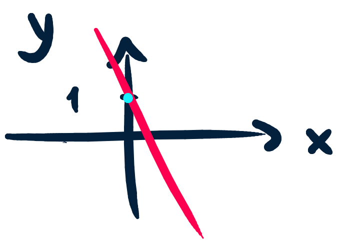


$f(x) = -x$

DECRESCENTE : $a(-)$



$f(x) = -2x$



$f(x) = -3x + 1$

EXEMPLO 01

Corrida de táxi $\begin{cases} \cdot \text{ R\$ 5,00 (fixo)} \\ \cdot \text{ R\$ 2,00 por km} \end{cases}$

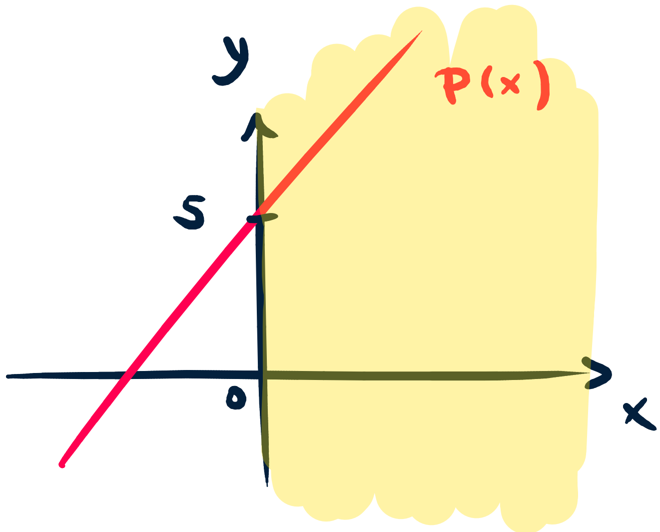
Quanto custa uma corrida de 10 km .

R :

P(x) → P : Preço

↳ x : nº de km

$$\boxed{P(x) = 2x + 5}$$

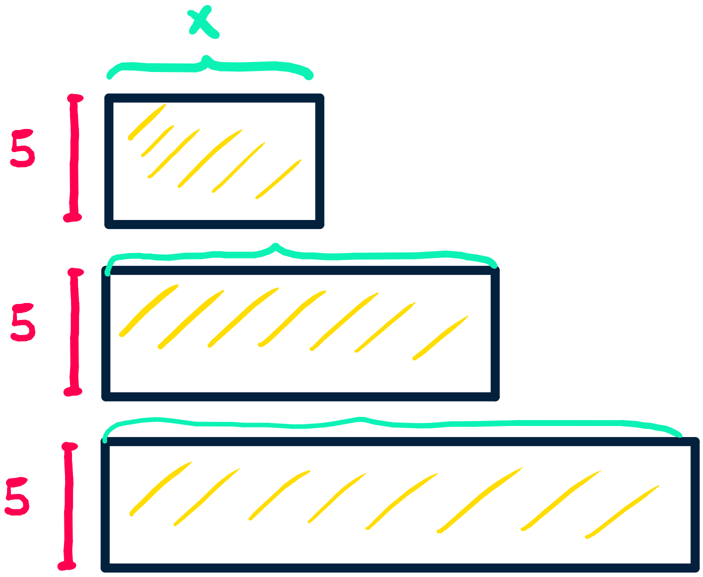


$P(10) = 2 \cdot 10 + 5 = 25$

↳ R\$ 25,00

EXEMPLO 02

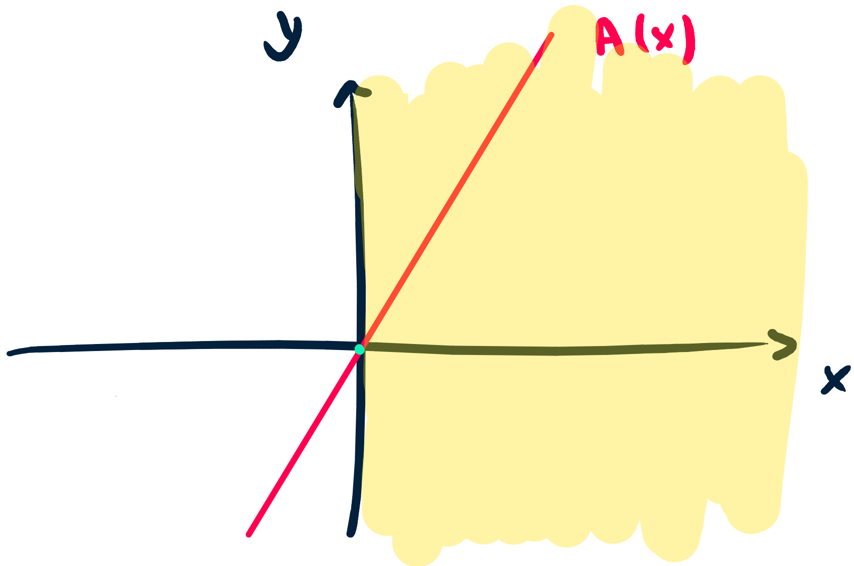
ÁREA DO RETÂNGULO : QUAL É A FUNÇÃO ?



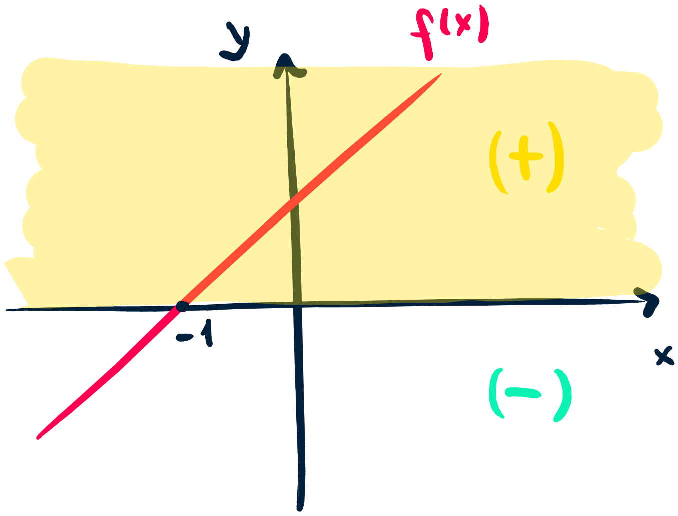
R :

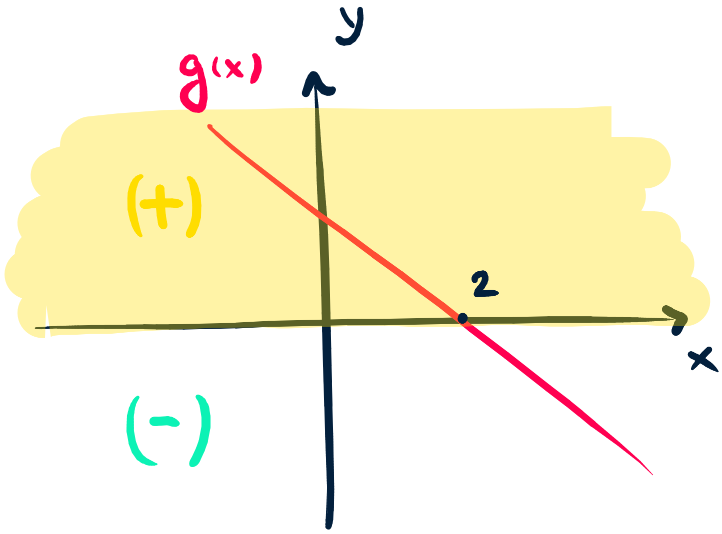
$$A = b \cdot h = 5 \cdot x$$

$$\boxed{A(x) = 5 \cdot x}$$



SINAL DE UMA FUNÇÃO $\to y = f(x)$





$f(x) > 0$ p/ $x > -1$

$g(x) > 0$ p/ $x < 2$

EXEMPLO

Determine quando a função $f(x) = 5x - 10$ assume valores negativos.

R:

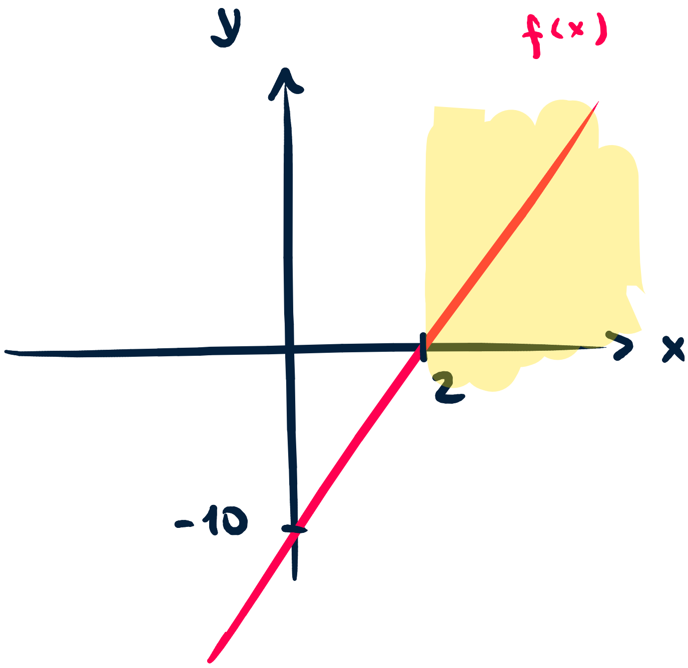
$f(x) = 5x - 10$

$\hookrightarrow f(x) = 0$

$5x - 10 = 0$

$5x = 10$

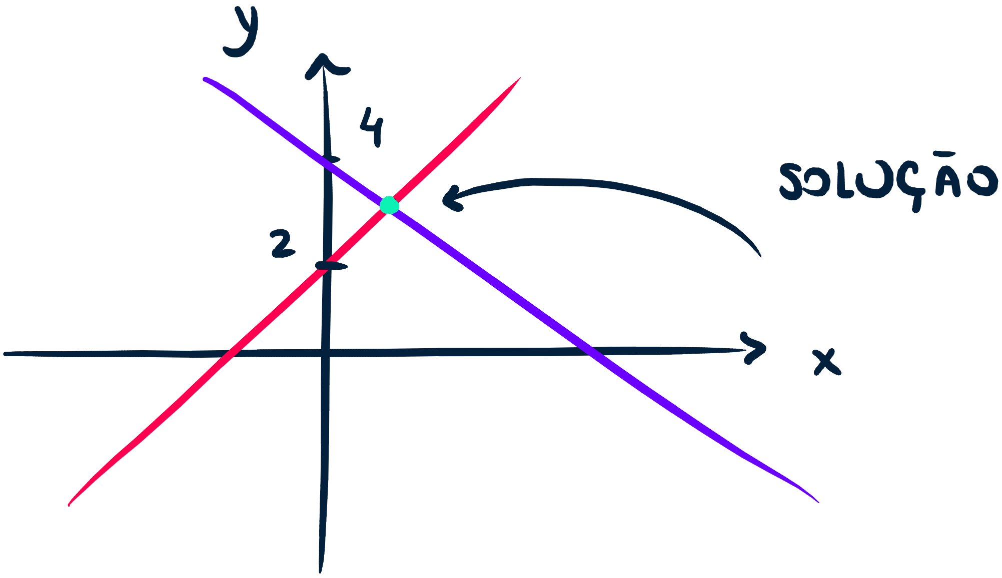
$x = 2$



R: $f(x) < 0$ para $x < 2$

EQUAÇÕES DO 1º GRAU

$$\begin{cases} x - y = -2 \dashrightarrow y = x + 2 \\ x + y = 4 \dashrightarrow y = -x + 4 \end{cases}$$



EXEMPLO 01

RESOLVA O SISTEMA:

$$\begin{cases} 2x - y = 6 \\ x + y = 3 \end{cases}$$

R:

i) SOLUÇÃO ALGÉBRICA

$$\begin{cases} 2x - y = 6 \\ x + y = 3 \end{cases} \quad (+)$$

$$3x = 9 \quad \therefore \quad x = 3$$

$$x + y = 3 \quad \therefore \quad 3 + y = 3 \quad \therefore \quad y = 0$$

Solução: $(x, y) = (3, 0)$

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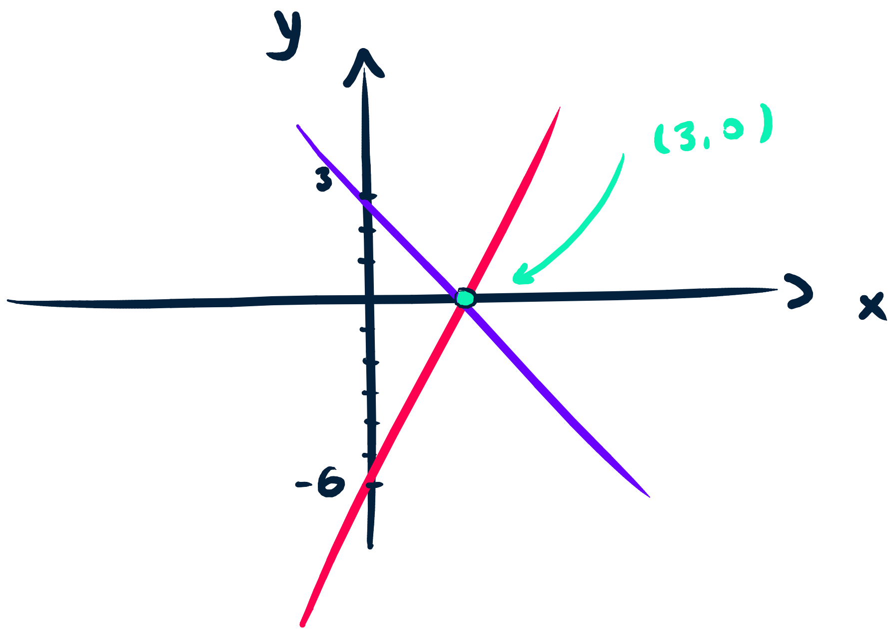
ii) INTERPRETAÇÃO GEOMÉTRICA

$$\begin{cases} 2x - y = 6 & \dashrightarrow \quad y = 2x - 6 \\ x + y = 3 & \dashrightarrow \quad y = -x + 3 \end{cases}$$



EXEMPLO 02

RESOLVA O SISTEMA:

$$\begin{cases} 2x - y = -6 \\ 4x - 2y = -20 \end{cases}$$

R:

i) SOLUÇÃO ALGÉBRICA

$$\begin{cases} 2x - y = -6 \xrightarrow{\times 2} 4x - 2y = -12 \\ 4x - 2y = -20 \longrightarrow 4x - 2y = -20 \end{cases}$$

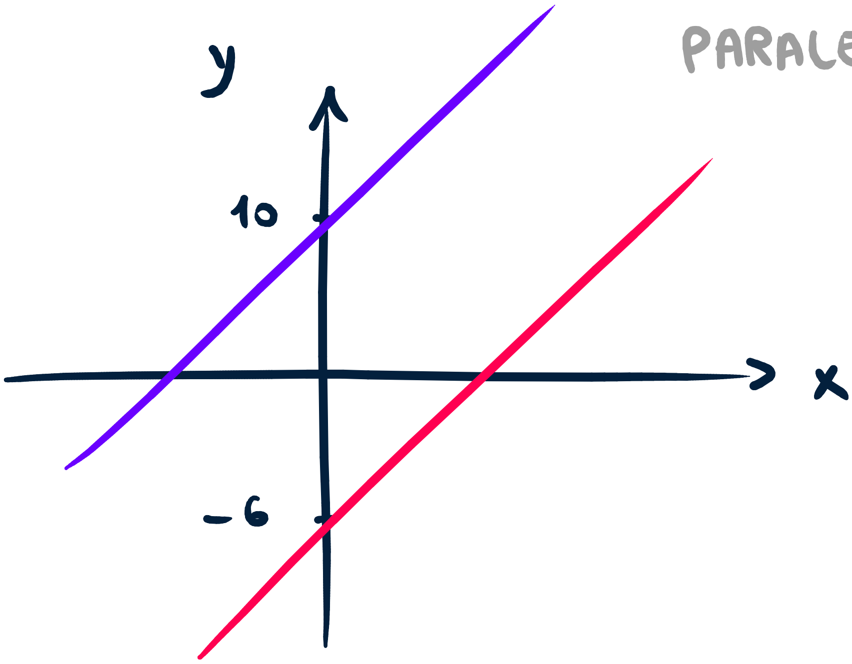
$$-20 = -12$$

$$\cancel{5 = 3}$$

NÃO TEM SOLUÇÃO

ii) INTERPRETAÇÃO GEOMÉTRICA

$$\begin{cases} 2x - y = -6 & \dashrightarrow \quad y = \underline{\underline{2}}x - 6 \\ 4x - 2y = -20 & \dashrightarrow \quad y = \underline{\underline{2}}x + 10 \end{cases}$$



PARALELAS

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INEQUAÇÕES DO 1º GRAU

$f(x) > g(x)$ $f(x) < g(x)$

$f(x) \geq g(x)$ $f(x) \leq g(x)$

EX.:

$$2x - 3 > x + 4$$

$f(x) = 2x - 3$

$g(x) = x + 4$

EXEMPLO 01

RESOLVA A INEQUAÇÃO $2x > x + 2$

R.:

i) SOLUÇÃO ALGÉBRICA

$$2x > x + 2$$

$$2x - 2 > x + 2 - 2$$

$$2x - 2 > x$$

$$2x - 2 - x > x - x$$

$$x - 2 + 2 > 0 + 2$$

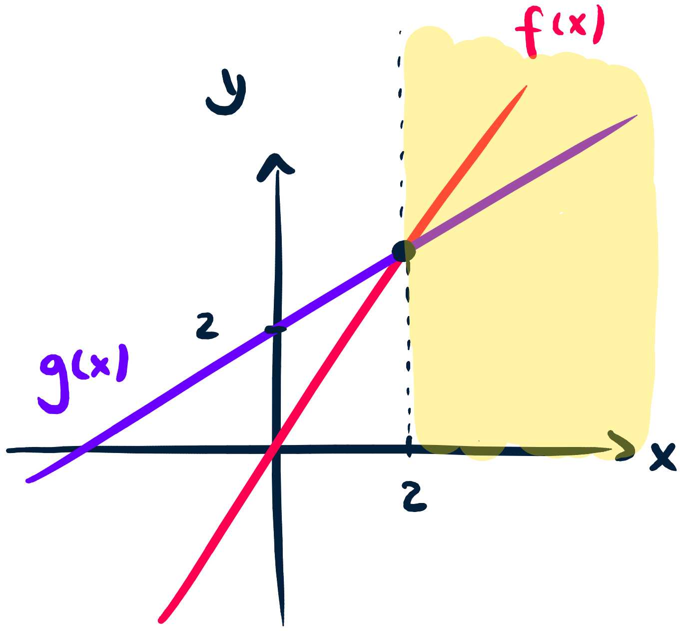
$$\boxed{x > 2}$$

ii) INTERPRETAÇÃO GEOMÉTRICA

$$2x > x + 2$$

$f(x) = 2x$ $\qquad$ $g(x) = x + 2$

$$f(x) > g(x)$$



EXEMPLO 02

RESOLVA A INEQUAÇÃO $x - 1 \leq 2x - 4$

R.:

i) SOLUÇÃO ALGÉBRICA

$$x - 1 \leq 2x - 4$$

$$x - 1 + 4 \leq 2x$$

$$x + 3 \leq 2x$$

$$3 \leq 2x - x$$

$$\boxed{x \geq 3}$$

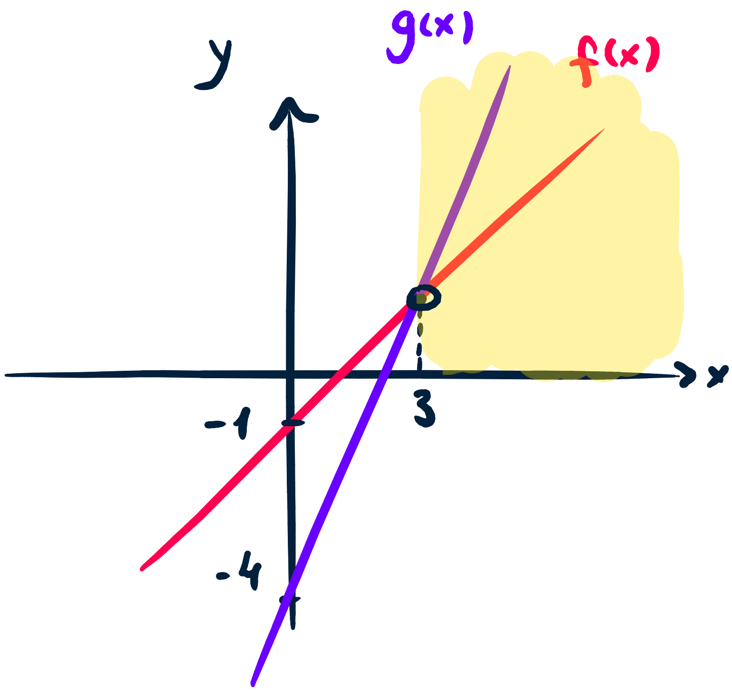
ii) INTERPRETAÇÃO GEOMÉTRICA

$$x - 1 \leq 2x - 4$$

$f(x) = x - 1$

$g(x) = 2x - 4$

$$f(x) \leq g(x)$$



EXEMPLO 03

RESOLVA A INEQUAÇÃO $-2x > 10$

R.:

i) SOLUÇÃO ALGÉBRICA

$$-2x > 10$$

$$x < \frac{10}{-2}$$

$$\boxed{x < -5}$$

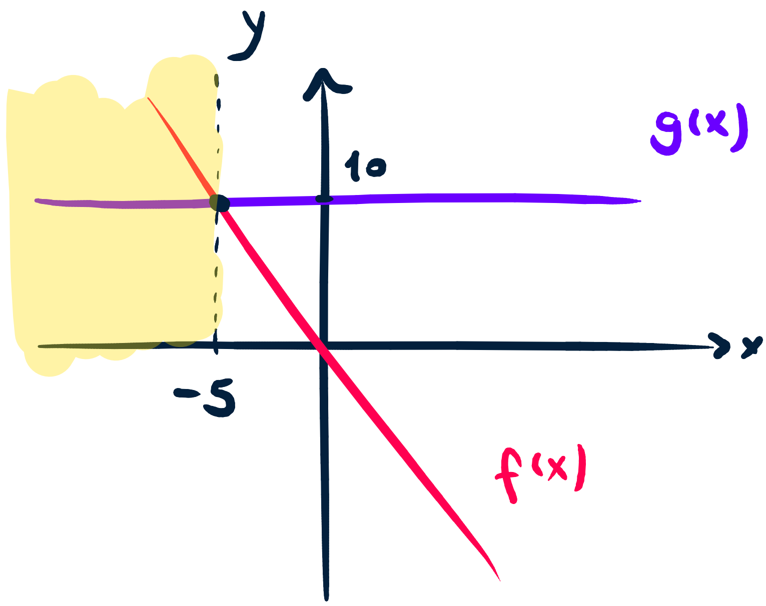
ii) INTERPRETAÇÃO GEOMÉTRICA

$$-2x > 10$$

$f(x) = -2x$

$g(x) = 10$

$$f(x) > g(x)$$



EXEMPLO 04

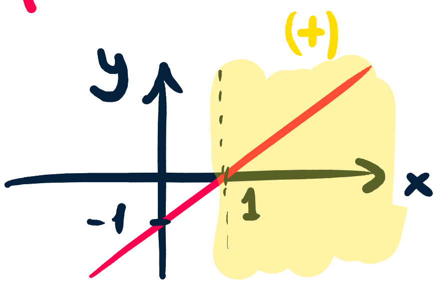
RESOLVA A INEQUAÇÃO $\underbrace{(x-1)}_{f(x)} \cdot \underbrace{(x+2)}_{g(x)} > 0$

$f(x) \cdot g(x) > 0$

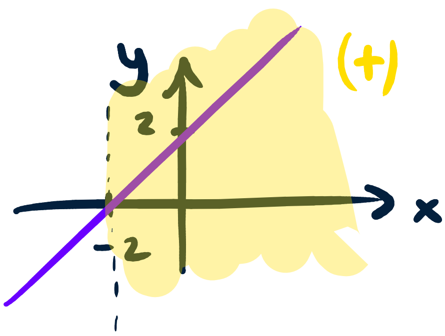
R.:

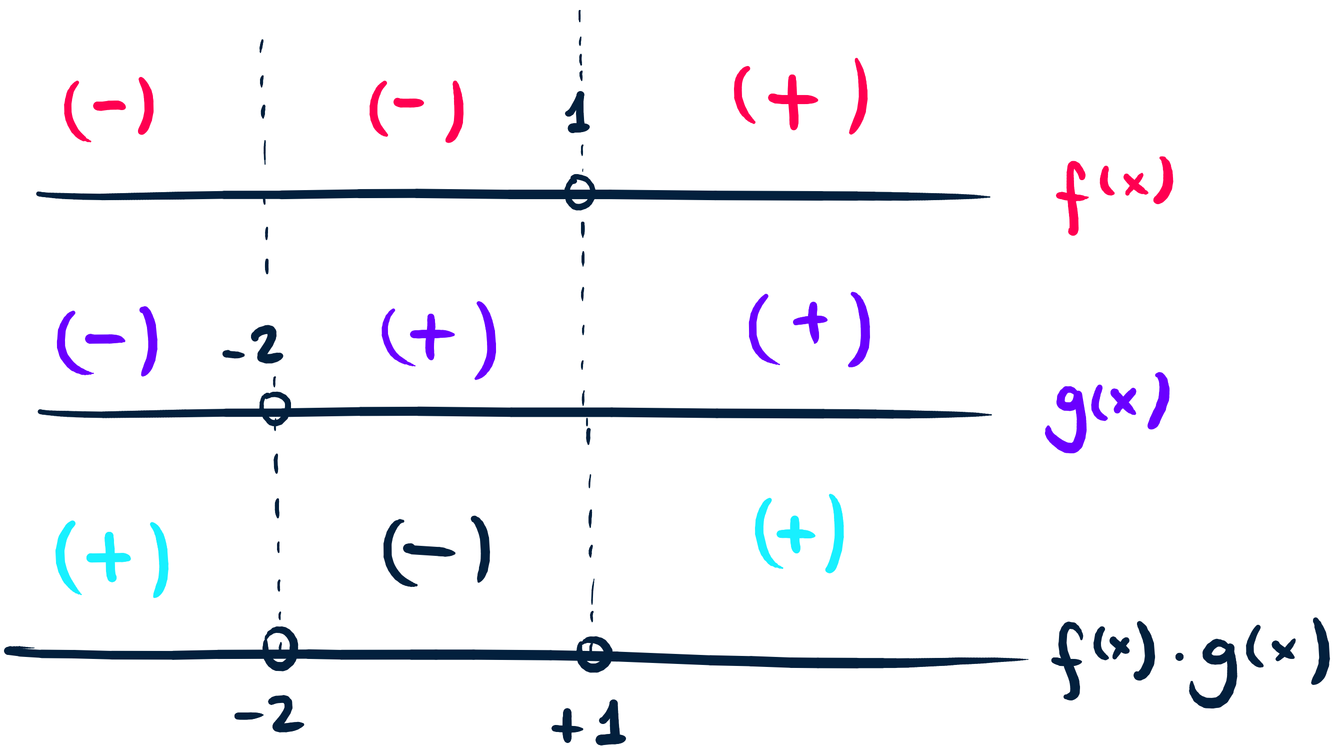
OBS.: QUADRO DE SINAIS

$f(x) = x - 1$



$g(x) = x + 2$





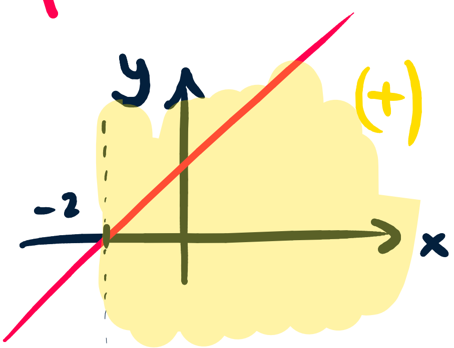
Sol.: $x < -2$ ou $x > 1$

EXEMPLO 05

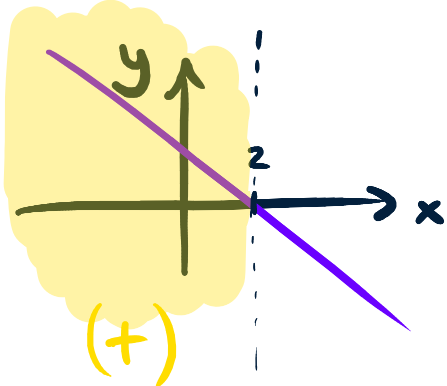
RESOLVA A INEQUAÇÃO $\dfrac{(2x+4)}{(-x+2)} \leq 0$

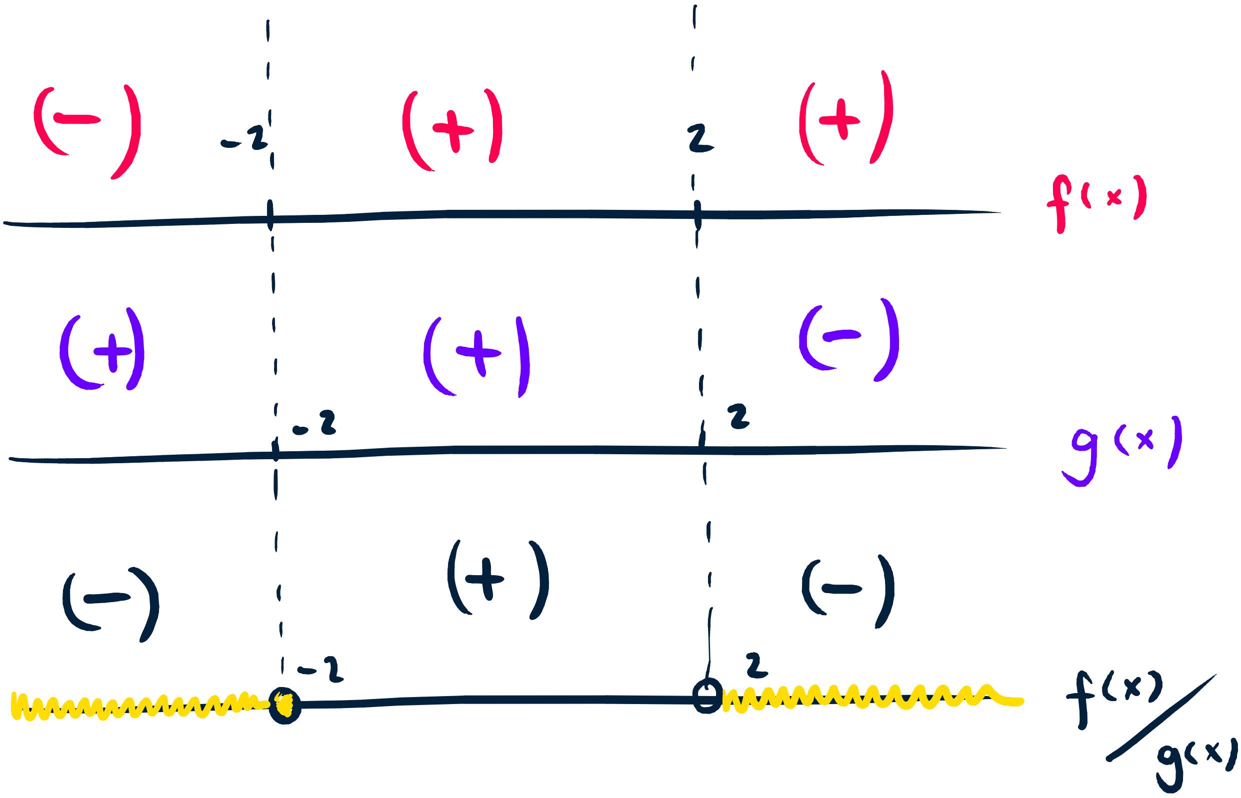
R.:

$f(x) = 2x + 4$



$g(x) = -x + 2$





Sol.: $x \leq -2$ ou $x > 2$